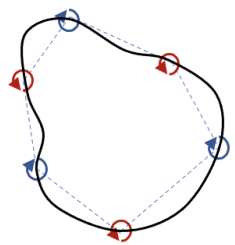
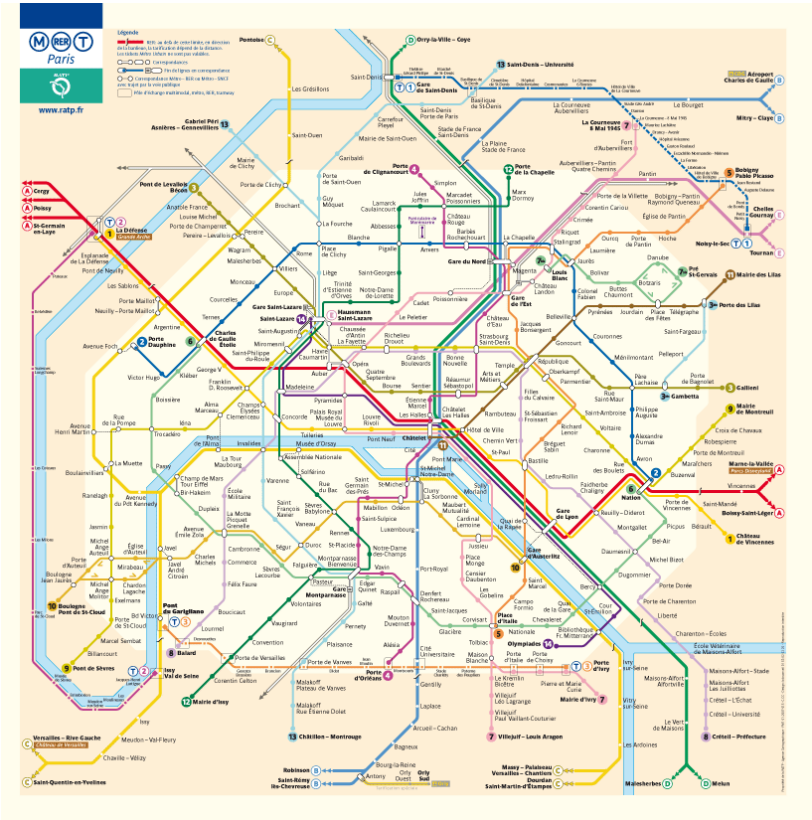
Graph theory analysis of topological structure

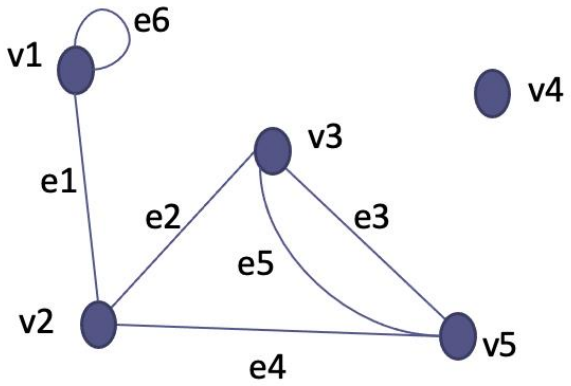


Xin Li 2021-05-14

graph theory



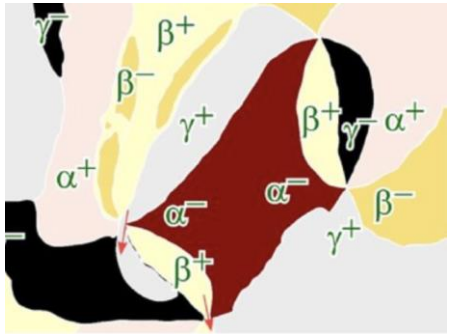
what is graph ? A set of points and lines joining these points.



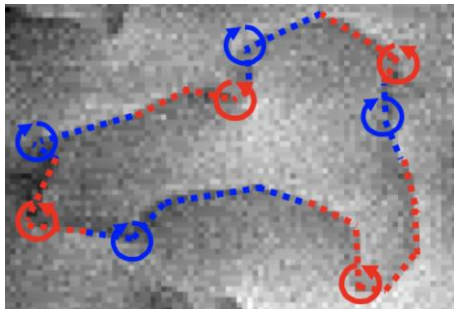
G=(V,E), V-vertices, E-edges

Topological defects

graphs as models :mathematical structures of pairwise relations between objects.

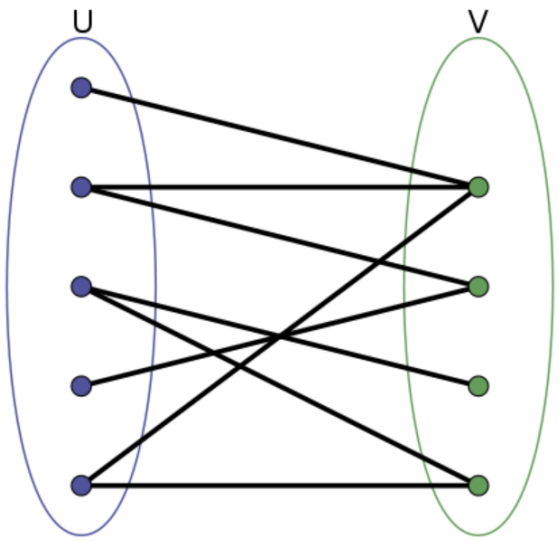


FE vortice & antivortice pair



pair of screw dislocations

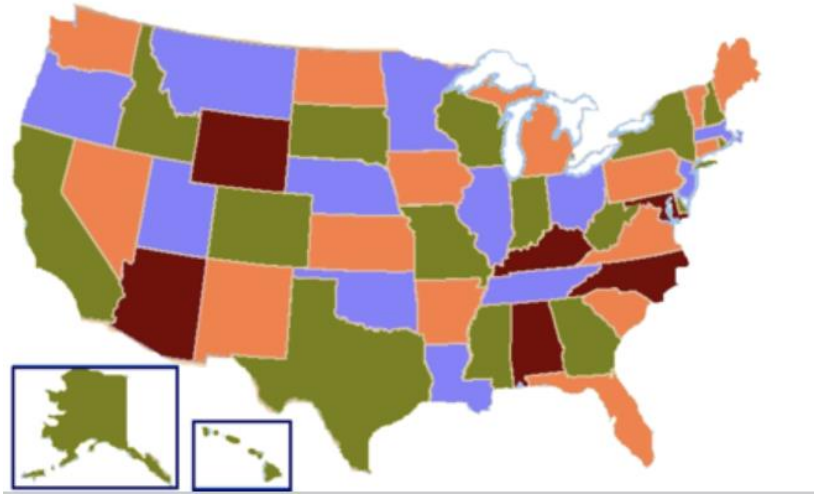
bipartite graph



vertices can be divided into
two disjointed and independent set
of U and V

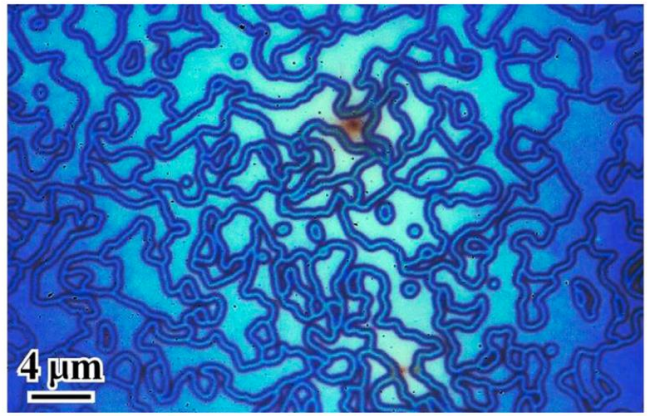
Graph coloring

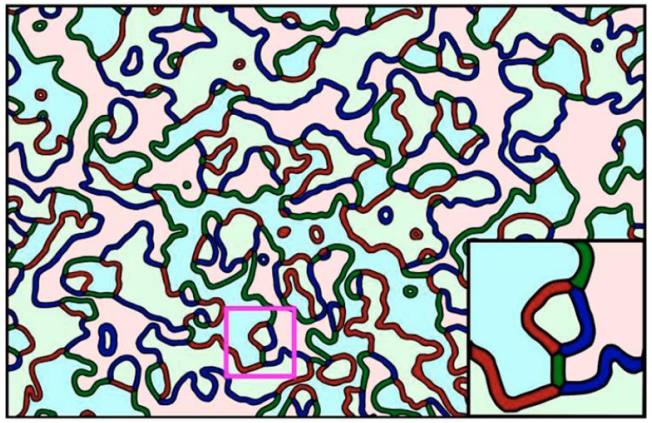
coloring a graph so that no two adjacent vertices (edges) have the same color.



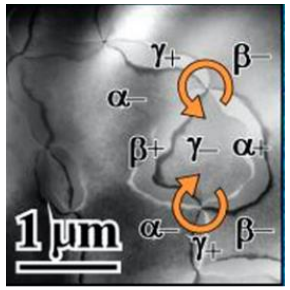
four color theorem

FE vortice and antivortice





Ferroelectric domain pattern and coloring of the pattern in $YMnO_3$



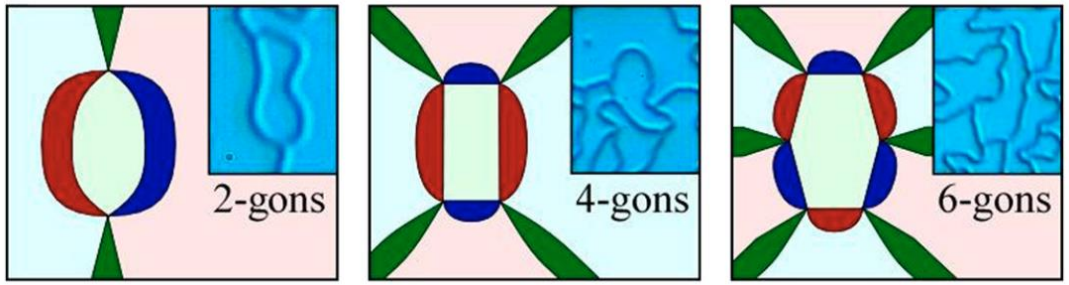


Table 1. The number of colors for proper coloring of three- and six-valent planar graphs whose faces (or domains) are all even gons

Valence	Vertex coloring	Edge coloring	Face coloring
3	2	3	3
6	2	6	2

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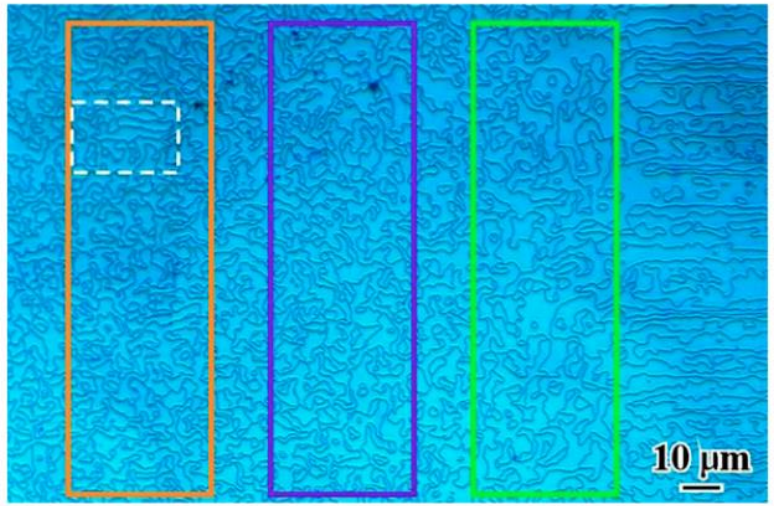
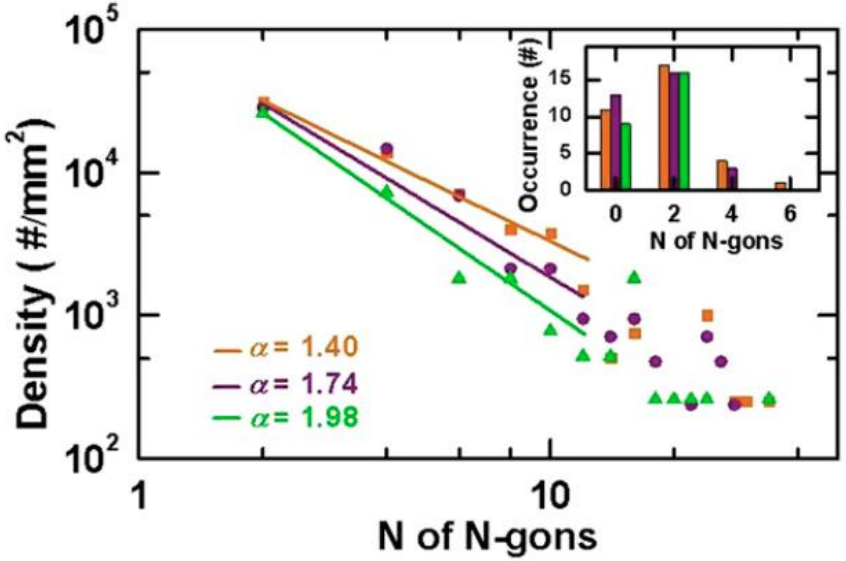
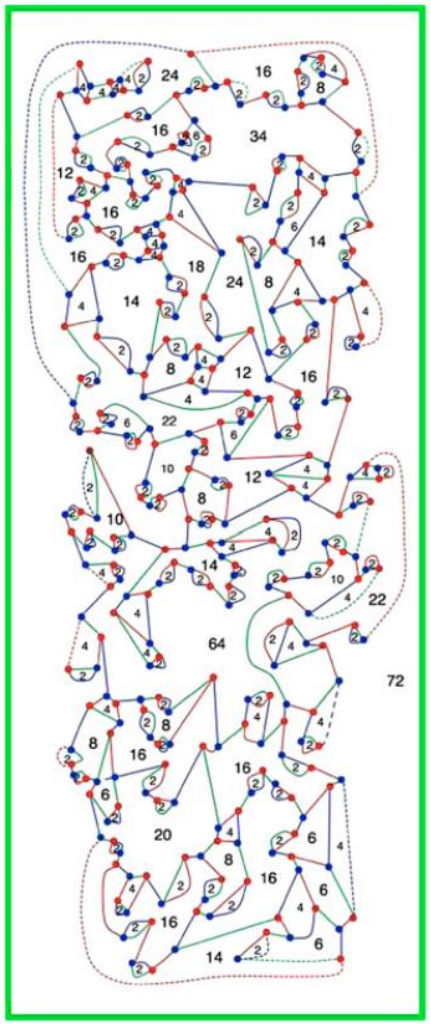
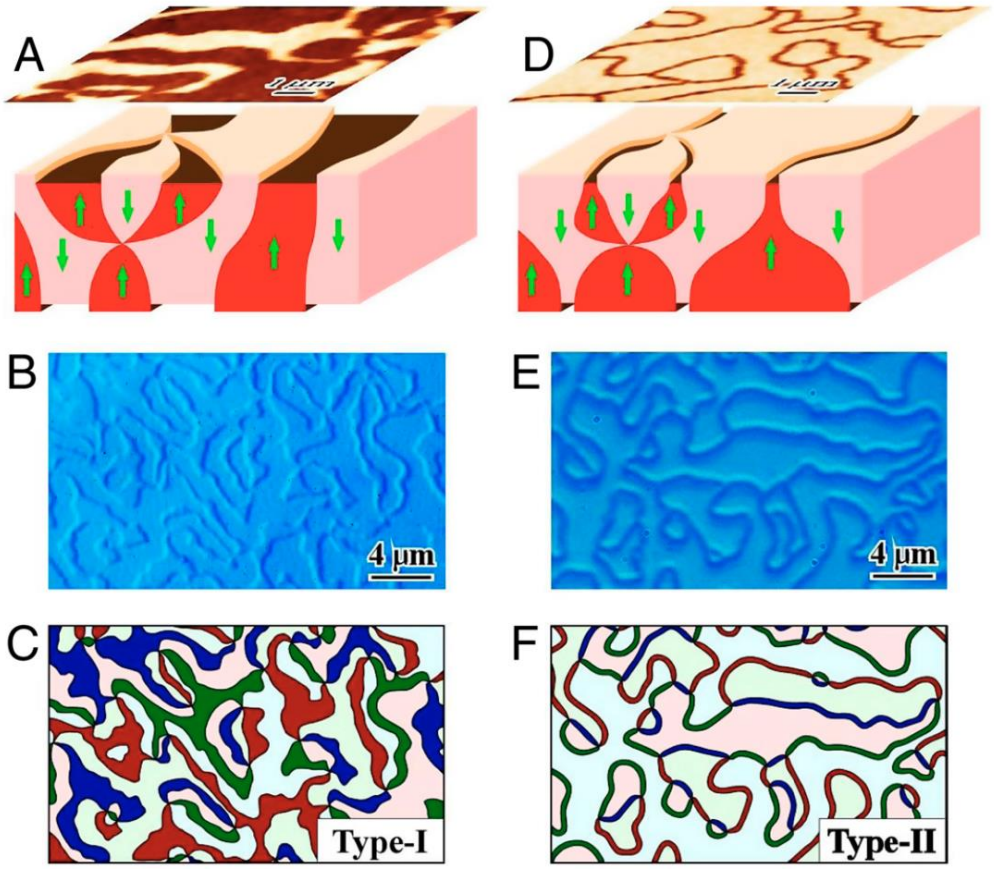
The power-law distribution of even-gons







Two types of the configuration of copious topological vortices and antivortices

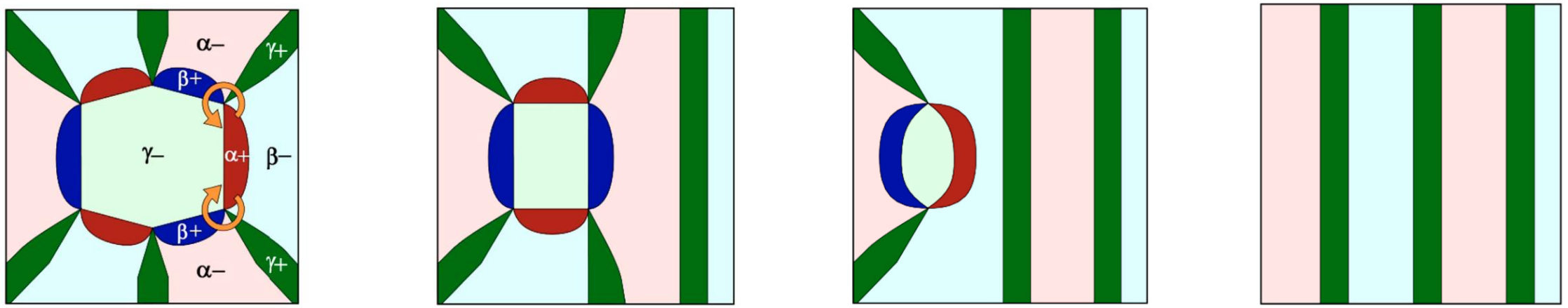


even-gons + domains
& even gons - domains

→

2-gons narrow + domains
& even gons broad- domains

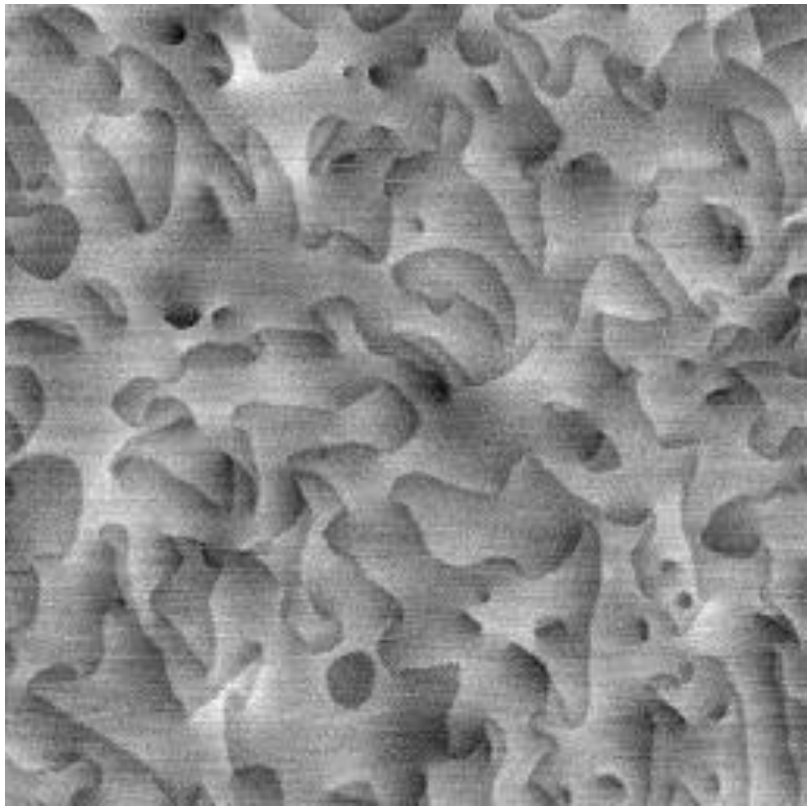
annihilation of vortex–antivortex pairs

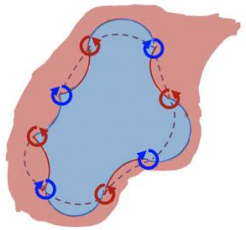
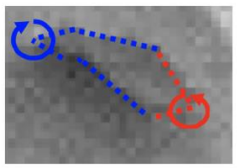
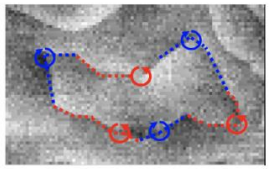
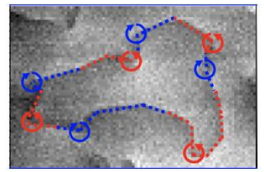
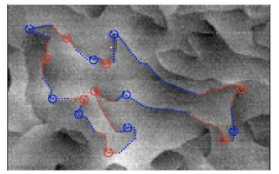
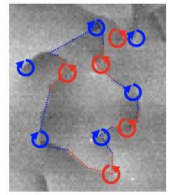


annihilation

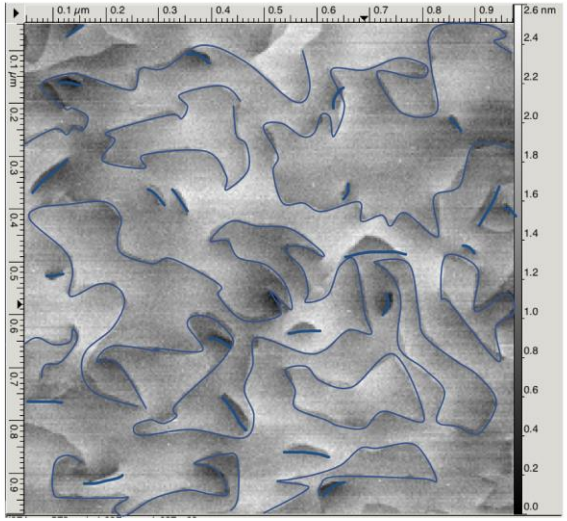
Sequential decomposition of a high-gon into lower-gons

Pair of screw dislocations in h-$ScFeO_3$

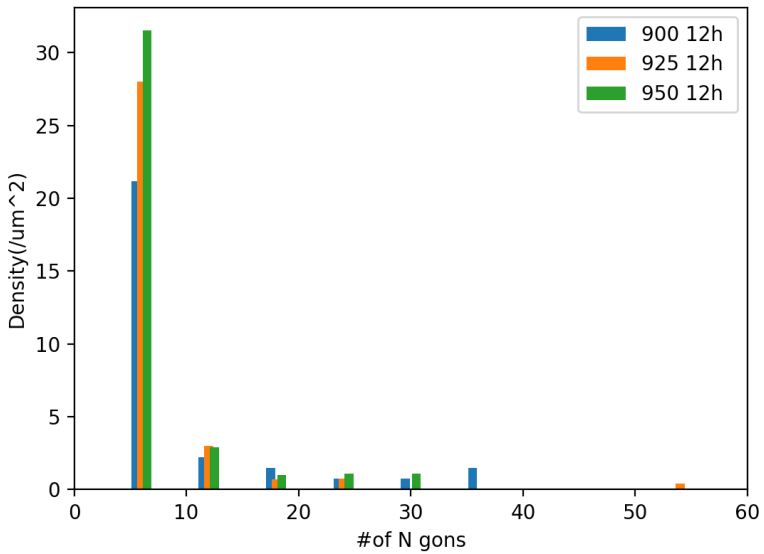


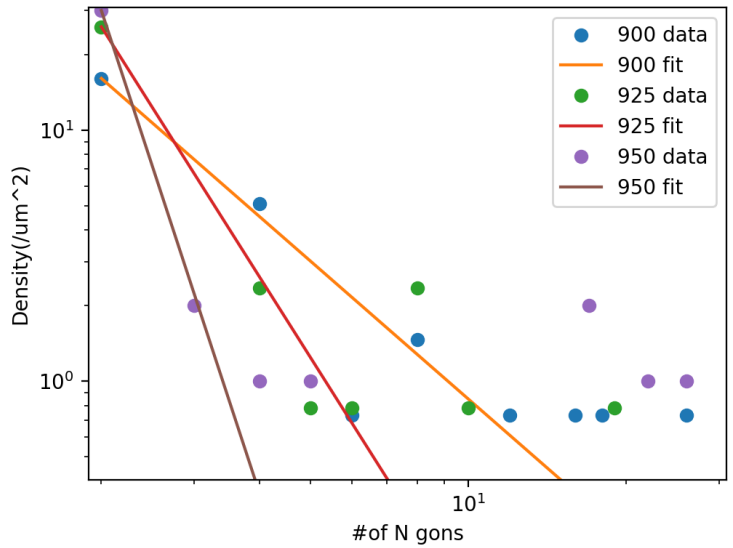
					
Notes number	2	6	8	16	.....
Closed surface					
					

YY239 950 12h 0.927*0.927 um



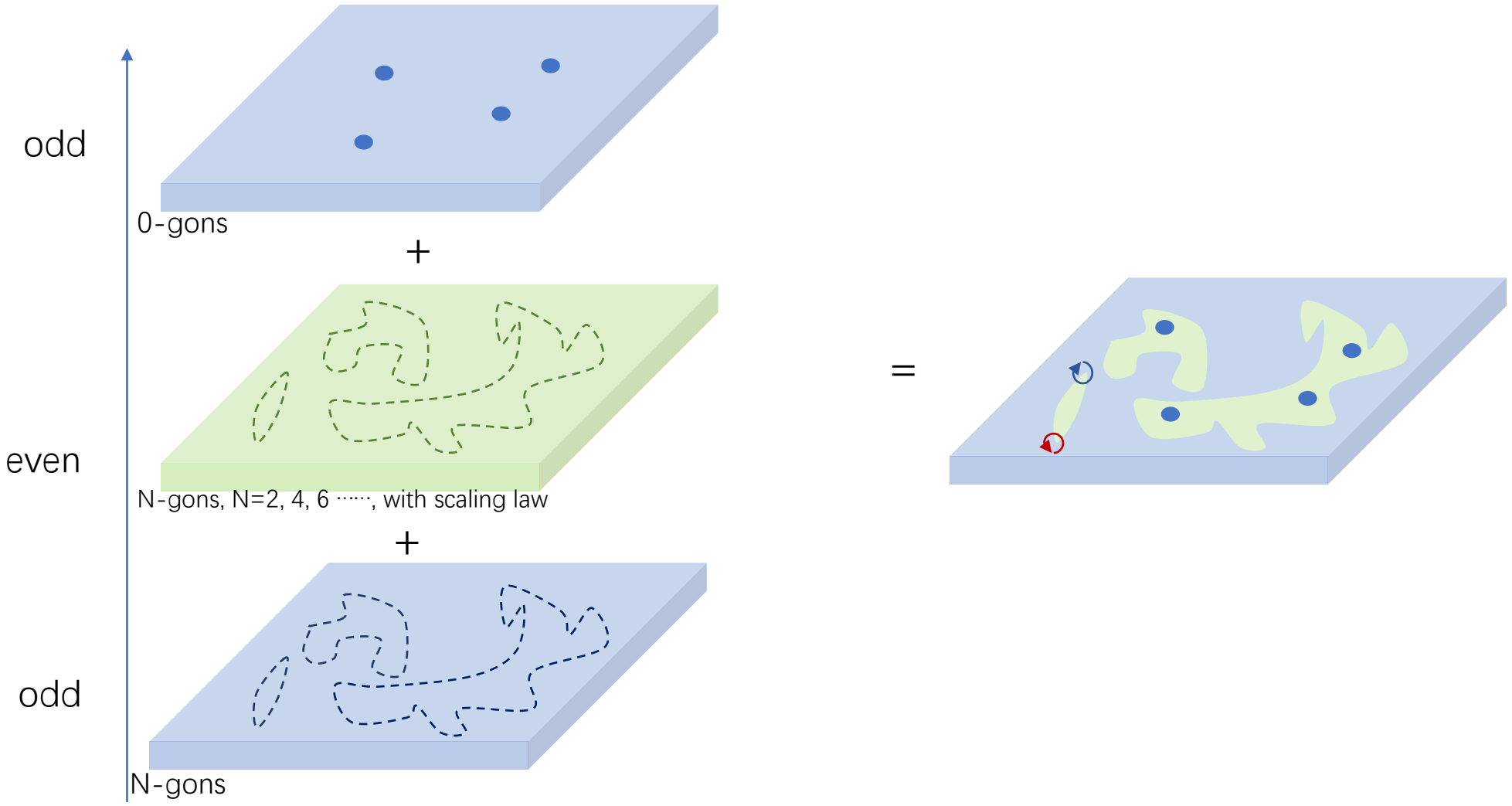
N-gons	number
2	22
4	2
5	1
6	3
8	1
10	1
18	1
24	1



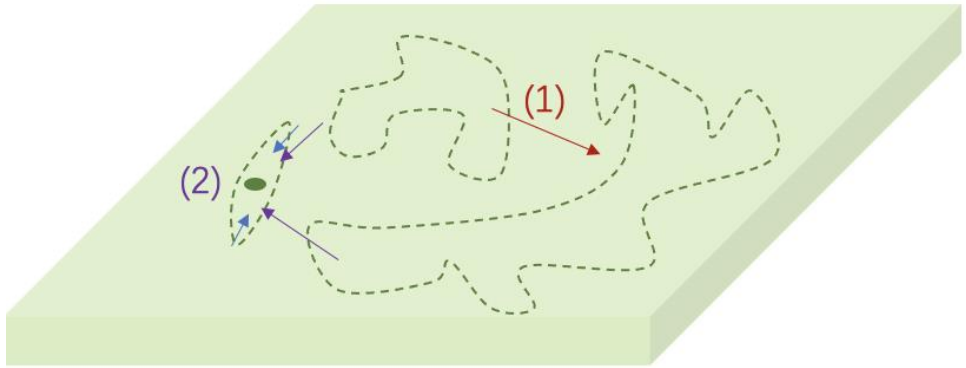


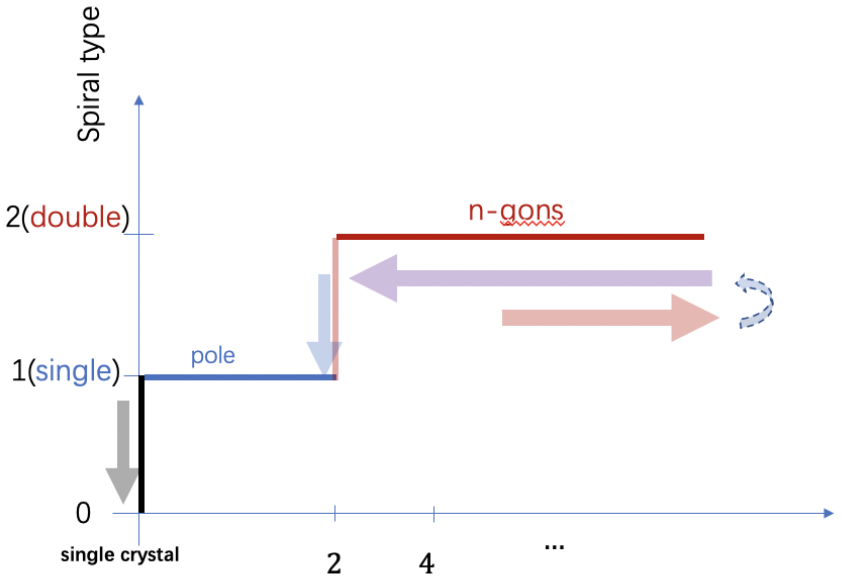
The power-law distribution of even-gons, similar to FE vortices and antivortices pair

thermal driven annihilation of screw dislocation pairs
similar to electrical poling driven annihilation of FE vortices and antivortice pair



$$N_{odd} = 0 + N\ (\sim\infty) = \sum g_i\, N_i = N_{even}$$





annihilation

(1) several N-gons coalescence

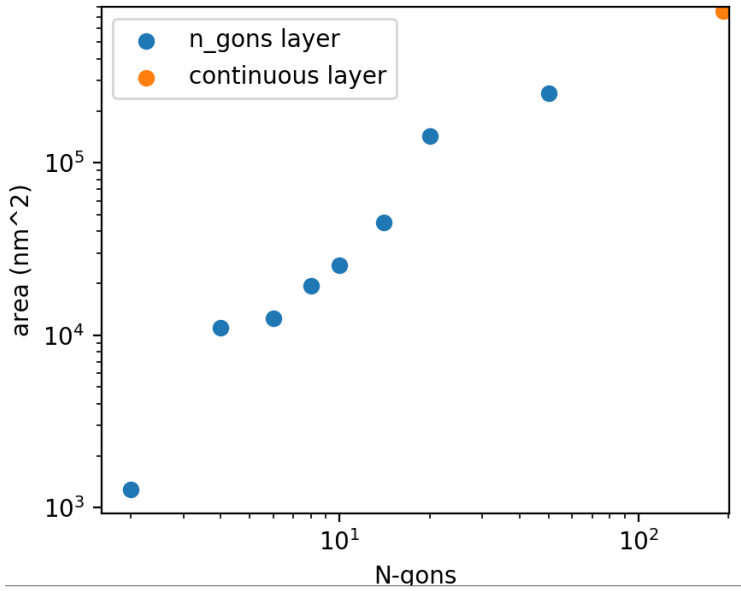
(2) Isloated N-gons shrink (N to $\frac{N}{2}$ *to* $\frac{N}{4}$ ….*to* 2)

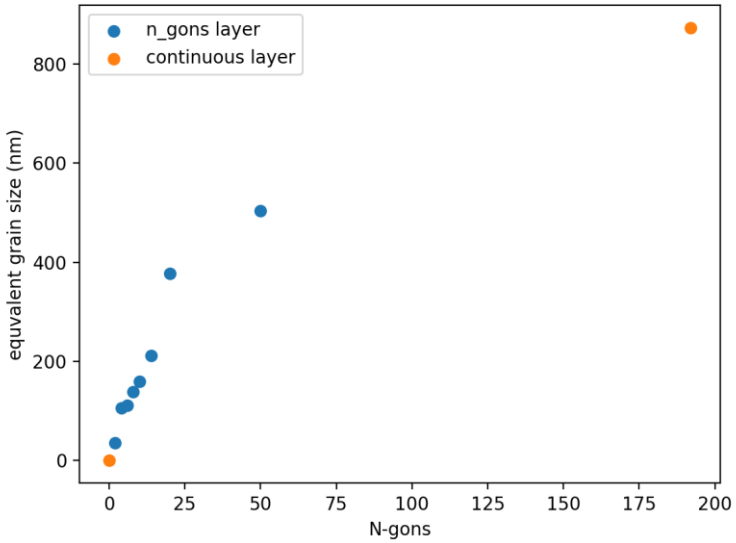
double spiral

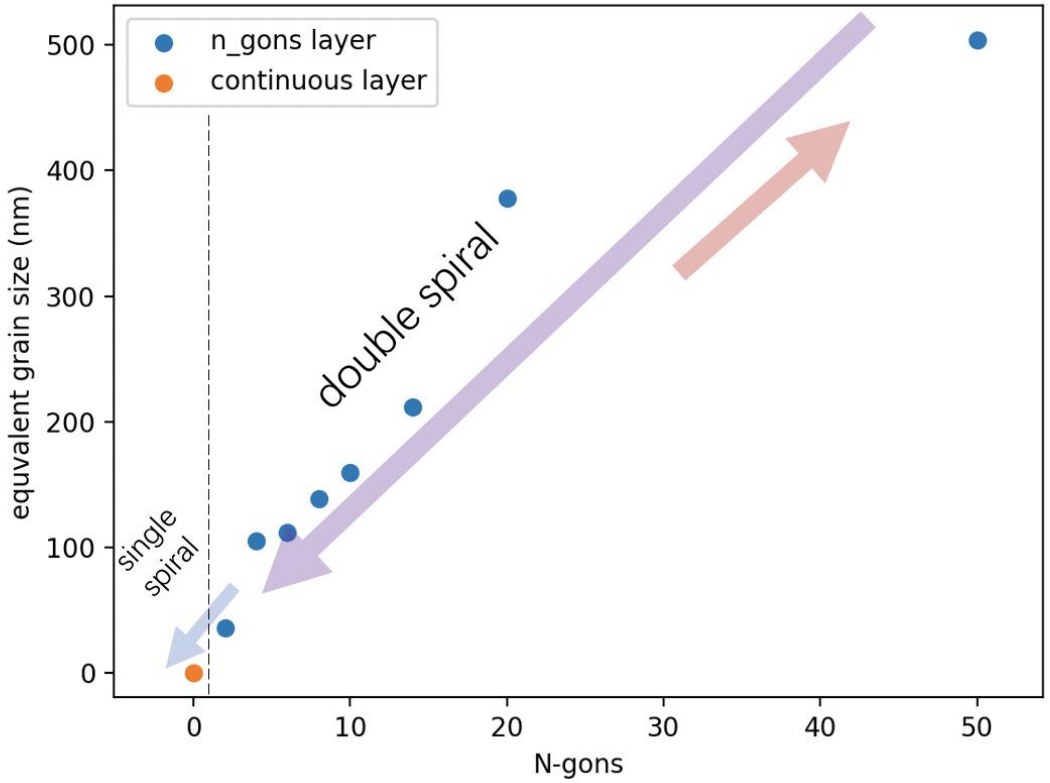
(3) 2-gons shrink to 0-gons

single spiral

(4) 0-gons annihilation


n_gons layer
continuous layer
area (nm^2)
10^5
10^4
10^3
10^1
10^2
N-gons


n_gons layer
continuous layer
equivalent grain size (nm)
800
600
400
200
0
0
25
50
75
100
125
150
175
200
N-gons


n_gons layer
continuous layer
equivalent grain size (nm)
500
400
300
200
100
0
double spiral
single spiral
0
10
20
30
40
50
N-gons

Summary:

1.Graph theory analysis can be used to quantify large-scale topological structures.

2. Power-law distribution of even gons reflects the annihilation level .