Theory of helical magnetic spin structure

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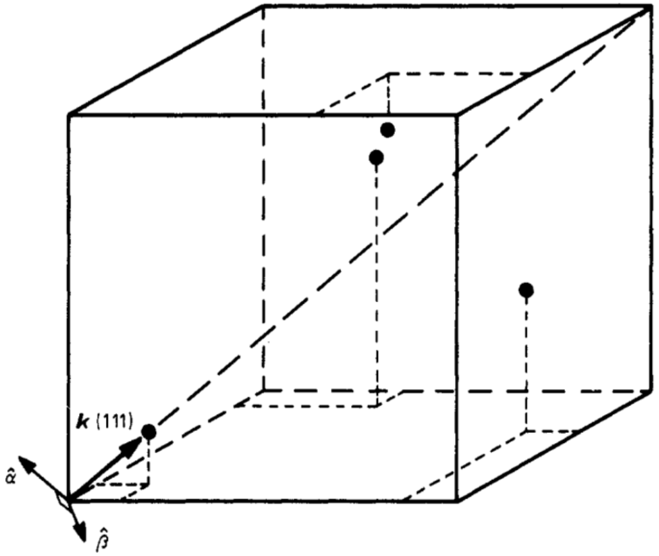


Crystal structure of B20 magnet ($P2_13$). There are four metal atoms in the positions (x, x, x), (x + 1/2, 1/2 - x, -x), (-x,x + 1/2,1/2 - x) and (1/2 - x, -x, x + ½). no inversion symmetry

symmetry of $P2_13$ structure

SYMM1	x,y,z
SYMM2	x+1/2,-y+1/2,-z
SYMM3	-x,y+1/2,-z+1/2
SYMM4	-x+1/2,-y,z+1/2
SYMM5	z,x,y
SYMM6	z+1/2,-x+1/2,-y
SYMM7	-z,x+1/2,-y+1/2
SYMM8	-z+1/2,-x,y+1/2
SYMM9	y,z,x
SYMM10	y+1/2,-z+1/2,-x
SYMM11	-y,z+1/2,-x+1/2
SYMM11	-y+1/2,-z,x+1/2

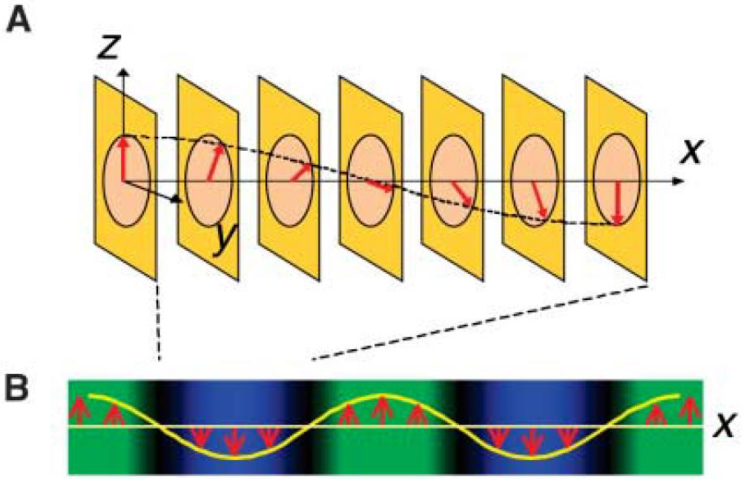


Illustration of helical spin order. (A) Helical spin order with the helical axis along the x axis. (B) Magnetization distribution projected on the xy plane for this helical spin order.

Landau free energy

$$F = A(\vec{s})^2 + b\vec{s}(\nabla \times \vec{s}) + B_1\left[(\nabla s_x)^2 + \left(\nabla s_y\right)^2 + (\nabla s_z)^2\right] + B_2(\nabla\vec{s})^2$$

$$= A\left({s_x}^2 + {s_y}^2 + {s_z}^2\right) + b\vec{s}(\nabla \times \vec{s}) + B_1\left[(\nabla s_x)^2 + \left(\nabla s_y\right)^2 + (\nabla s_z)^2\right]$$

$$+ B_2\left[\left(\frac{\partial s_x}{\partial x}\right)^2 + \left(\frac{\partial s_y}{\partial y}\right)^2 + \left(\frac{\partial s_z}{\partial z}\right)^2\right]$$

The free energy is minimized by periodic structure

$$\vec{s} = \frac{1}{\sqrt{2}}\left[\overrightarrow{S_k}\exp\left(i\vec{k}\vec{r}\right) + \overrightarrow{S_k}^{\,*}\exp\left(-i\vec{k}\vec{r}\right)\right]$$

$\vec{k}$ is the propagation vector

Determine the direction of $\vec{k}$

$$F = \mathrm{A}\left|\overrightarrow{S_k}\right|^2 + \boxed{ib\vec{k}\left(\overrightarrow{S_k} \times \overrightarrow{S_k}^{\,*}\right)} + B_1\vec{k}^2\left|\overrightarrow{S_k}\right|^2 + \boxed{B_2\left(k_x^2|S_{kx}|^2 + k_y^2\left|S_{ky}\right|^2 + k_z^2|S_{kz}|^2\right)}$$

setting $\vec{S}_k = \vec{a}_k + i\vec{b}_k$, above equation is minimized when choosing $\vec{a}_k \perp \vec{b}_k$ and $|\vec{a}_k| = \left|\vec{b}_k\right|$

$ib\vec{k}\left(\overrightarrow{S_k} \times \overrightarrow{S_k}^{\,*}\right)$= $ib\vec{k}(\vec{a}_k + i\vec{b}_k) \times (\vec{a}_k - i\vec{b}_k)$

= $\mathrm{i}b\vec{k}(-i\vec{a}_k \times \vec{b}_k + i\vec{b}_k \times \vec{a}_k + \vec{a}_k \times \vec{a}_k + \vec{b}_k \times \vec{b}_k)$

= $2b\vec{k}\vec{a}_k \times \vec{b}_k$

$\vec{k} \parallel \vec{a}_k \times \vec{b}_k$ for b<0, left-handed

$\vec{k} \parallel -\vec{a}_k \times \vec{b}_k$ for b>0, right hand

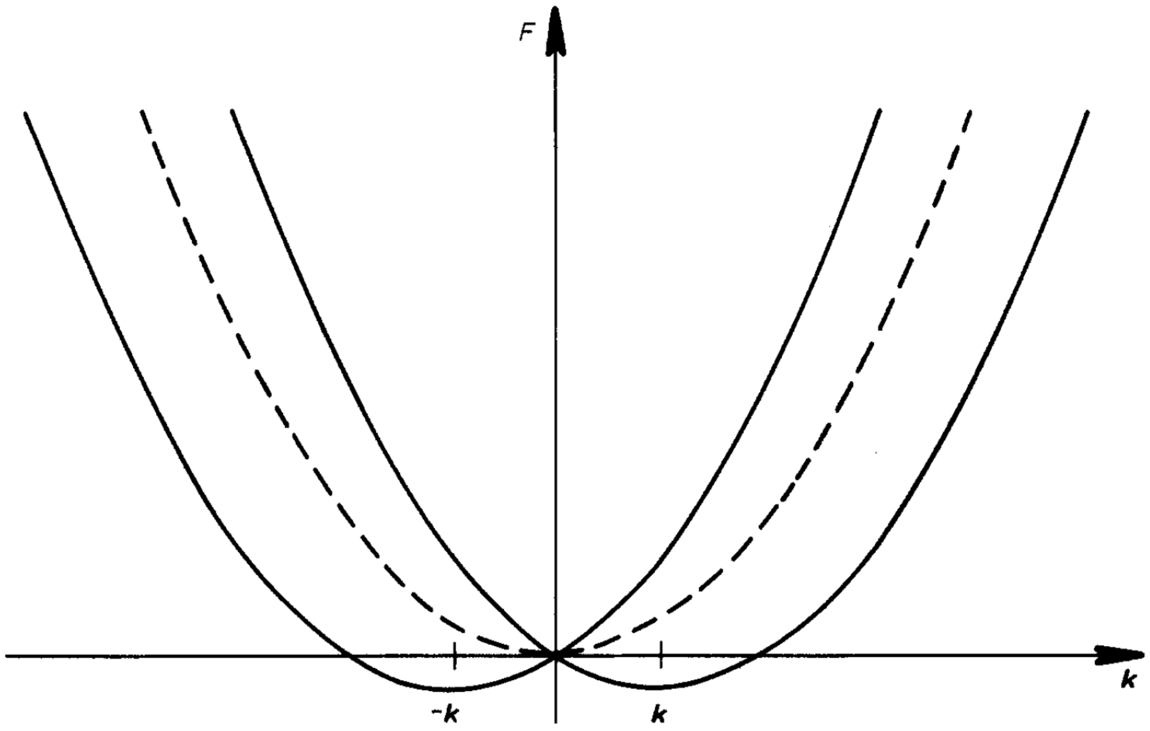
The helical spin structure arise from no inversion symmetry (DM interaction)

For example, B20 material $\vec{k} \parallel (111)$

$$F = (A - bk)\left|\overrightarrow{S_k}\right|^2 + \left(B_1 + \tfrac{1}{3}B_2\right)k^2\left|\overrightarrow{S_k}\right|^2$$

For min:

$$k = \frac{b}{2B_1 + \frac{2}{3}B_2}, B_1 + \frac{1}{3}B_2 > 0$$



Free energy as a function of wavevector for left-handed and right-handed spirals.
The broken curve shows the free energy for a system with inversion symmetry (***b*** = 0).

$$real\ part\ of$$

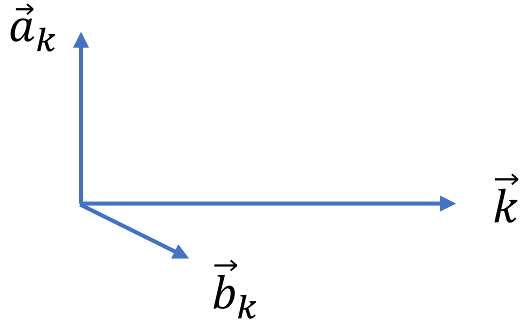
$$\vec{s} = \frac{1}{\sqrt{2}}\left[\overrightarrow{S_k}\exp\left(i\vec{k}\vec{r}\right) + \overrightarrow{S_k}^*\exp\left(-i\vec{k}\vec{r}\right)\right]$$

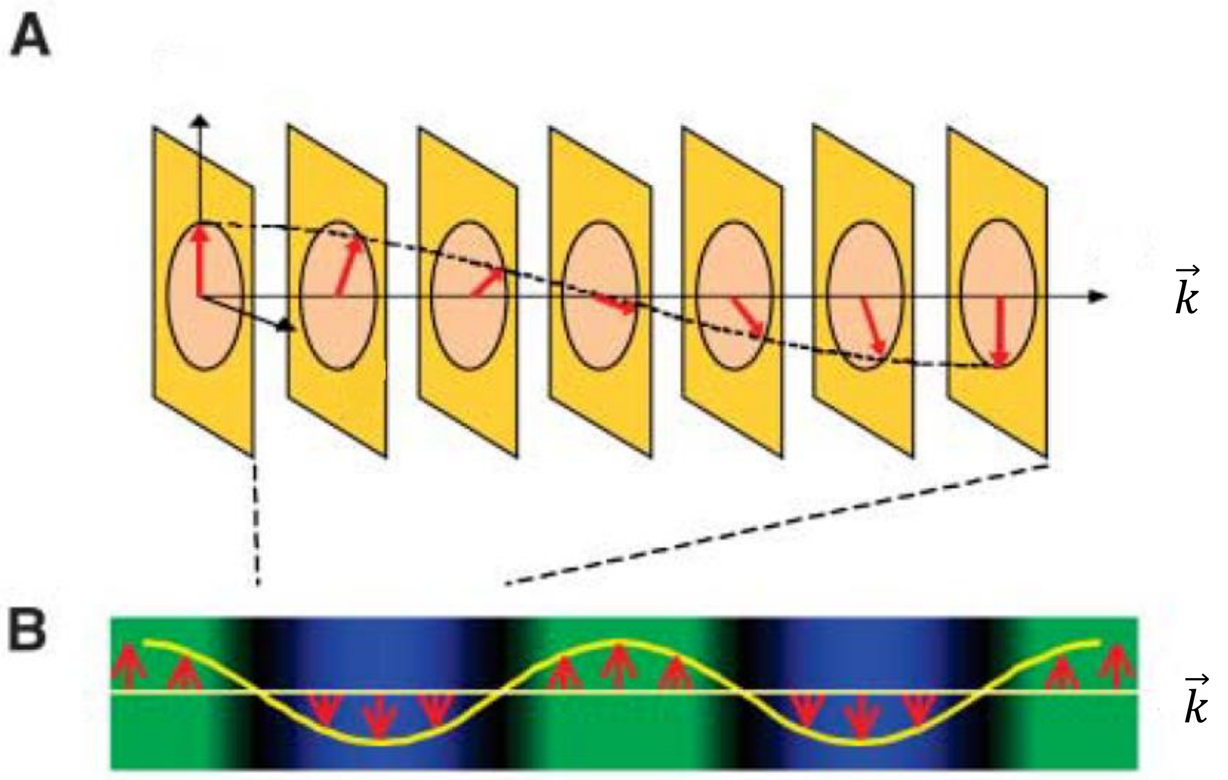
$$\vec{s} = \vec{a}_k\cos\left(\vec{k}\vec{r}\right) - \vec{b}_k\sin\left(\vec{k}\vec{r}\right)$$

$$\vec{s} = \vec{a}_k \cos(\vec{k}\vec{r}) - \vec{b}_k sin(\vec{k}\vec{r})$$

$\vec{k} \parallel \vec{a}_k \times \vec{b}_k$ for left-handed

$\vec{k} \parallel -\vec{a}_k \times \vec{b}_k$ for right hand





Conclusion

The helical spin structure comes from the DM interaction term in Landau free energy which is nonzero when there is no inversion symmetry.

Landau theory can only tell the relation between propagation vector and magnetic spin($\vec{k} \parallel \pm\vec{a}_k \times \vec{b}_k$), the specific details of spin can be discussed by other method like group theory analyze.