

Anisotropic magnetoresistance in ferromagnetic metals

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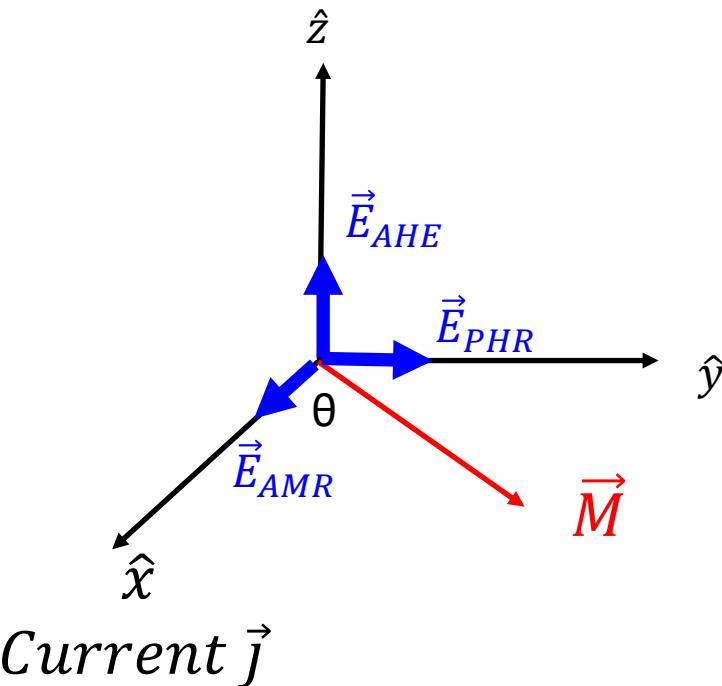
See Previous (2021/12/11)

Ref. IEEE TRANSACTIONS ON MAGNETICS, VOL. MAG-11, p1018, NO. 4, JULY
1975

AMR in context

$$\vec{E} = \rho_{\perp} \vec{j} + \hat{M} (\vec{j} \cdot \hat{M}) (\rho_{\parallel} - \rho_{\perp}) + \rho_H \hat{M} \times \vec{j}$$

Assume $\vec{j} = j \hat{x}$, \vec{M} in x-y plane.



$$E_x = \rho_{\perp} j + \cos^2 \theta (\rho_{\parallel} - \rho_{\perp}) j$$

*This is typical **AMR** (anisotropy magnetoresistance).*

$$E_y = \sin \theta \cos \theta (\rho_{\parallel} - \rho_{\perp}) j$$

*This is called **PHR** (planer Hall effect).*

$$E_z = \rho_H \sin \theta j$$

*This is called **AHE** (anomalous Hall effect).*

Crystalline conductivity
anisotropy

Semiclassical theory, Relaxation time τ

- The relaxation time depends on the scattering rate

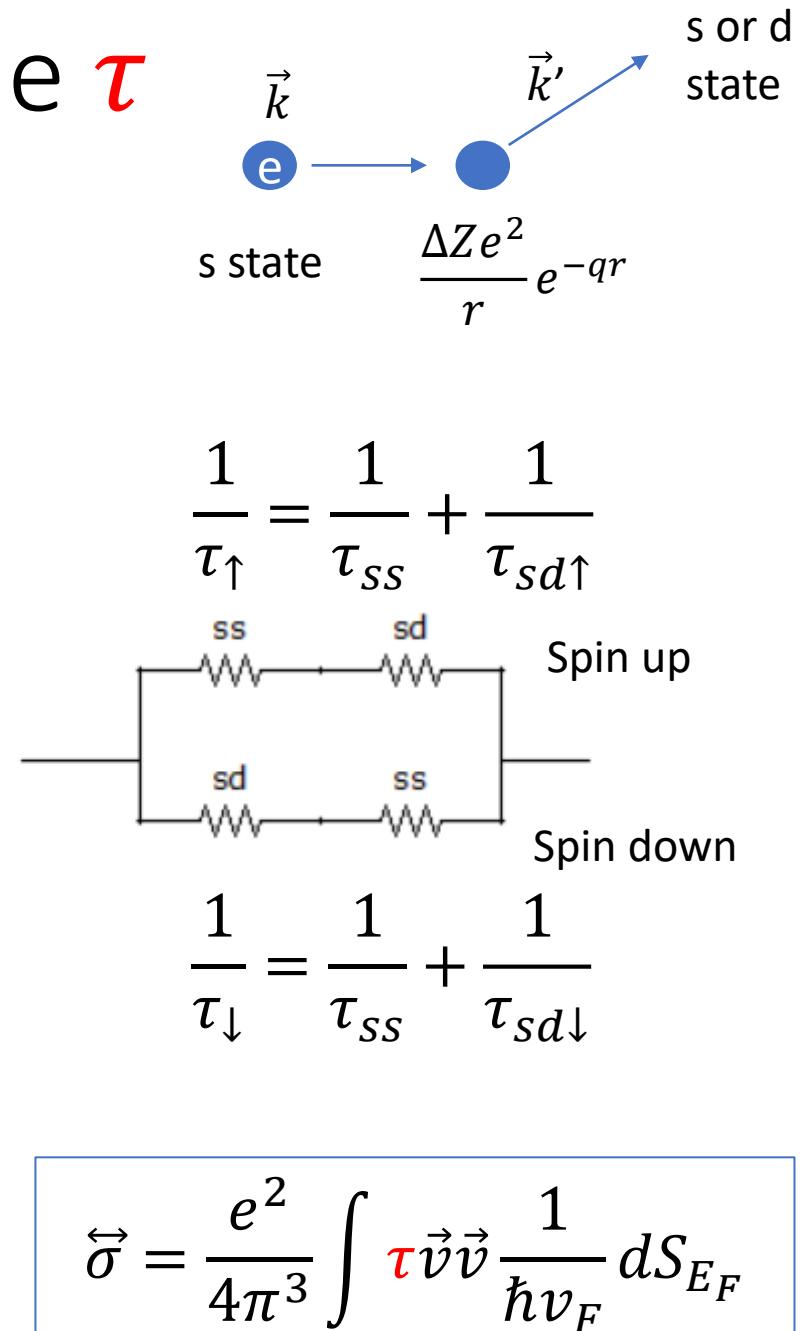
$$\frac{1}{\tau_{ss}} = \frac{2\pi}{\hbar} N_s(E_F) \int_0^\pi |\langle \psi_s | V_{scatt} | \psi_{s'} \rangle|^2 d\theta$$

$$\frac{1}{\tau_{sd}} = \frac{2\pi}{\hbar} N_d(E_F) |\langle \psi_s | V_{scatt} | \psi_d \rangle|^2$$

$\langle \psi_s | V_{scatt} | \psi_s \rangle$ and $|\langle \psi_s | V_{scatt} | \psi_d \rangle|$ are the transition matrix elements

- Scattering potential (screened coulomb)

$$V_{scatt} = \frac{\Delta Z e^2}{r} e^{-qr}$$



Calculating the scattering matrix

- ss scattering

$$\langle \psi_s | V_{scatt} | \psi_s' \rangle = \int e^{i\vec{k} \cdot \vec{r}} \frac{\Delta Z e^2}{r} e^{-qr} e^{i\vec{k}' \cdot \vec{r}} dr^3 = \frac{4\pi \Delta Z e^2}{(\vec{k} - \vec{k}')^2 + q^2}$$
$$\frac{1}{\tau_{ss}} = \frac{2\pi}{\hbar} N_s(E_F) \int_0^\pi |\langle \psi_s | V_{scatt} | \psi_s' \rangle|^2 d\theta = \frac{4(\pi \Delta Z e^2)^2}{q^2(2k_F^2 - q^2)}$$

θ : angle between \vec{k} and \vec{k}' . This is isotropic.

$$V_{scatt} = \frac{\Delta Z e^2}{r} e^{-qr}$$
$$\psi_s = e^{i\vec{k} \cdot \vec{r}}$$
$$\psi_{dm} = \phi_m(\vec{r})f(r)$$
$$f(r) \propto e^{-r \frac{z}{3a_0}}$$

- sd scattering $\phi_m(\vec{r}) = xy$ as an example.

$$\langle \psi_s | V_{scatt} | \phi_{xy} \rangle \propto \int \psi_s^* V_{scatt}(r) \textcolor{red}{xy} dr^3 = -32\pi \Delta Z e^2 \frac{\textcolor{red}{k_x k_y}}{(k^2 + q^2)^3} = \alpha \textcolor{red}{k_x k_y}$$

Similarly

$$\langle \psi_s | V_{scatt} | \phi_{x^2-y^2} \rangle \propto \int \psi_s^* V_{scatt}(r) (\textcolor{red}{x^2 - y^2}) dr^3 = -32\pi \Delta Z e^2 \frac{\textcolor{red}{k_x^2 - k_y^2}}{(k^2 + q^2)^3}$$
$$= \alpha (\textcolor{red}{k_x^2 - k_y^2})$$

This is anisotropic

Simplified case

$$\vec{\sigma} = \frac{e^2}{4\pi^3} \int \tau \vec{v} \vec{v} \frac{1}{\hbar v_F} dS_{E_F}$$

- *The incident is always s electron, which has parabolic (free electron gas) dispersion.*

$$\vec{v} = \frac{\hbar \vec{k}_F}{m}$$

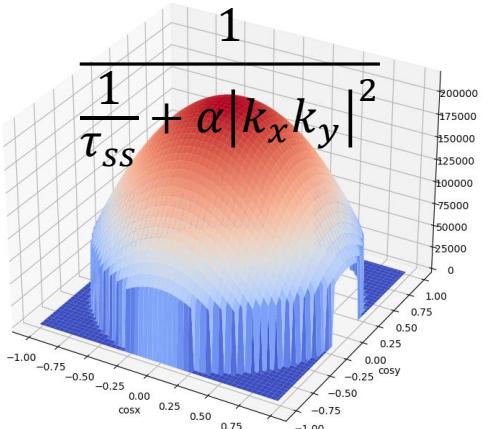
- *For an extreme case, consider only the d_{xy} state is on the Fermi surface*

$$\begin{aligned}\sigma_{ij} &= \frac{e^2 \hbar}{4\pi^3 m^2 v_F} \int \frac{1}{\frac{1}{\tau_{ss}} + \frac{1}{\tau_{sd}}} k_i k_j dS_{E_F} \\ &= \frac{e^2 \hbar}{4\pi^3 m^2 v_F} \int \frac{1}{\frac{1}{\tau_{ss}} + \frac{2\pi}{\hbar} N_d(E_F) |\alpha k_x k_y|^2} k_i k_j dS_{E_F}\end{aligned}$$

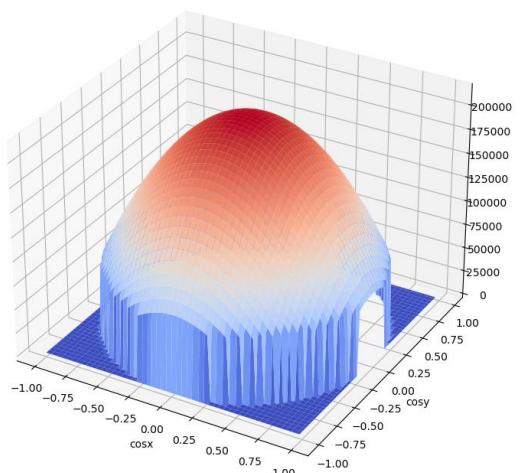
To visualize σ_{ij} , we plot the longitudinal conductivity

$$(\vec{\sigma} \cdot \hat{E}) \cdot \hat{E}$$

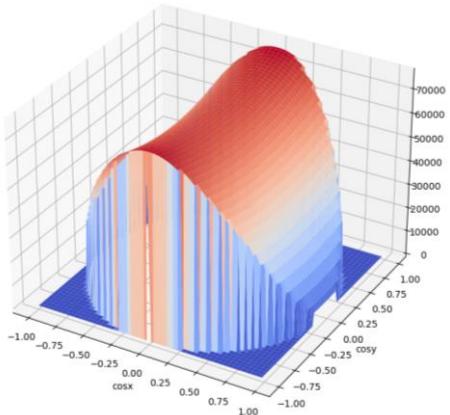
Examples



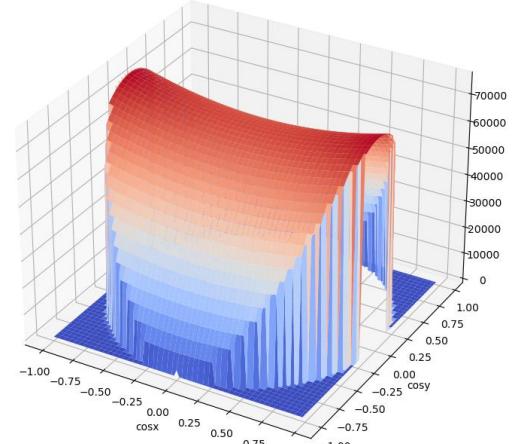
xy



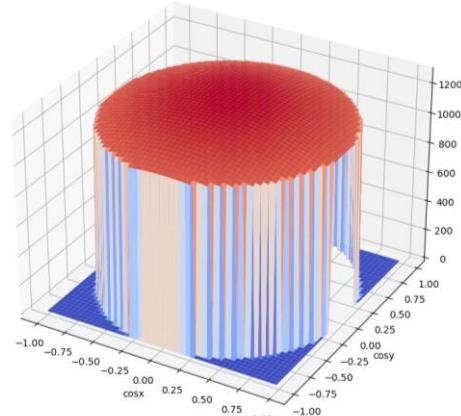
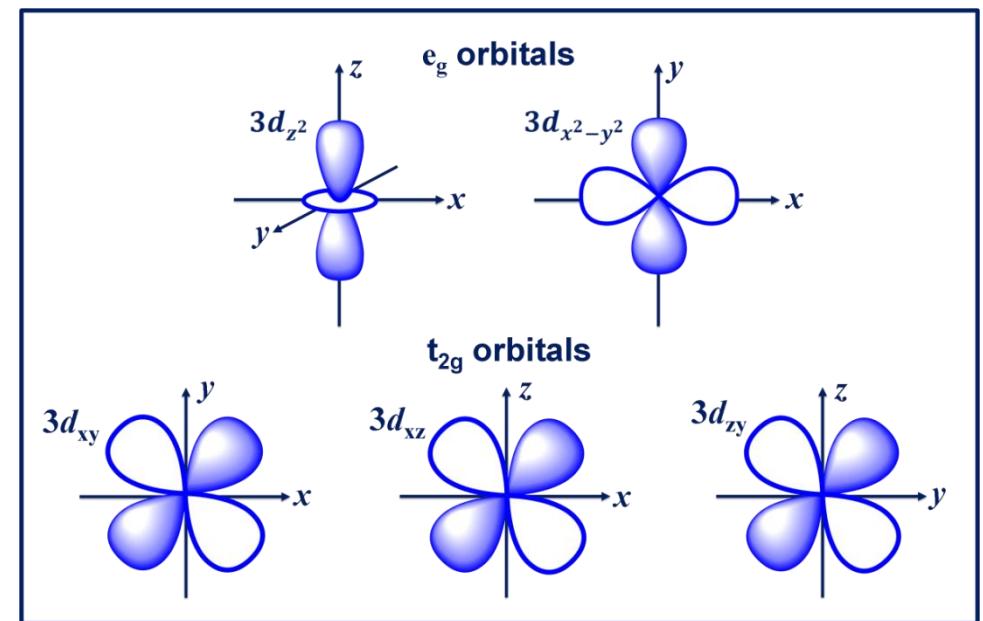
xx-yy



xz



yz



3zz-rr

Effect of spin direction on
Crystalline conductivity
anisotropy

Spin-orbit coupling

$$\frac{1}{\tau_{sd}} = \frac{2\pi}{\hbar} N_d(E_F) |\langle \psi_s | V_{scatt} | \psi_d \rangle|^2$$

- $\psi_{dm+}^1 = a_m \phi_m(\vec{r}) \chi_+ + \sum_n a_{mn+} \phi_n(\vec{r}) \chi_-$
- $\psi_{dm-}^1 = a_m \phi_m(\vec{r}) \chi_- + \sum_n a_{mn-} \phi_n(\vec{r}) \chi_+$

- Example: $x^2 - y^2$ state, spin along z direction

$$\begin{aligned} \langle x^2 - y^2 \uparrow | H_{SO} | xy \uparrow \rangle &= \frac{\xi}{2} (\langle 2 \uparrow | H_{SO} | 2 \uparrow \rangle - \langle -2 \uparrow | H_{SO} | -2 \uparrow \rangle) \\ &= \xi \end{aligned}$$

$$\langle x^2 - y^2 \uparrow | H_{SO} | xz \downarrow \rangle = \frac{1}{2} \langle -2 \uparrow | H_{SO} | -1 \downarrow \rangle = \frac{\xi}{2}. \quad H_{SO} = \xi$$

$$\langle x^2 - y^2 \uparrow | H_{SO} | yz \downarrow \rangle = \frac{1}{2} \langle -2 \uparrow | H_{SO} | -1 \downarrow \rangle = -\frac{\xi}{2}$$

Therefore,

$$\begin{aligned} |x^2 - y^2 \uparrow\rangle_1 &= \frac{1}{\sqrt{1 + \frac{3\xi^2}{2}}} \left[|x^2 - y^2 \uparrow\rangle + \xi |xy \downarrow\rangle + \frac{\xi}{2} (|xz \downarrow\rangle - |yz \downarrow\rangle) \right] \end{aligned}$$

$ -2, \downarrow \rangle$	$ -1, \downarrow \rangle$	$ 2, \uparrow \rangle$
1	0	0
0	$\frac{1}{2}$	0
0	0	0
0	0	$\frac{\sqrt{6}}{2}$
0	0	0
0	$-\frac{1}{2}$	0
0	0	0
0	0	$\frac{\sqrt{6}}{2}$
0	$\frac{\sqrt{6}}{2}$	0
0	0	0
0	$\frac{\sqrt{6}}{2}$	0
0	0	$\frac{1}{2}$
0	0	0

Spin along arbitrary direction xx-yy as example

$$|x^2 - y^2 \chi_+\rangle = \frac{1}{\sqrt{2}}(|2\rangle + |-2\rangle) \cos \frac{\theta}{2} (|\uparrow\rangle + \tan \frac{\theta}{2} e^{i\phi} |\downarrow\rangle)$$

$$\langle x^2 - y^2 \chi_+ | H_{SO} | xy \chi_+ \rangle = \xi$$

$$\langle x^2 - y^2 \chi_+ | H_{SO} | xz \chi_+ \rangle = \frac{\xi}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2} 2 \cos \phi = \frac{\xi}{2} \sin \theta \cos \phi$$

$$\langle x^2 - y^2 \chi_+ | H_{SO} | yz \chi_+ \rangle = -i \frac{\xi}{2} \sin \theta \sin \phi$$

$$\langle x^2 - y^2 \chi_+ | H_{SO} | xz \chi_- \rangle = \frac{\xi}{2} \cos^2 \frac{\theta}{2} \left(-\tan^2 \frac{\theta}{2} e^{-i2\phi} + 1 \right)$$

$$\langle x^2 - y^2 \chi_+ | H_{SO} | yz \chi_- \rangle = \frac{\xi}{2} \cos^2 \frac{\theta}{2} \left(-\tan^2 \frac{\theta}{2} e^{-i2\phi} - 1 \right)$$

Effect of spin orbit coupling

$$\frac{1}{\tau_{sd}} = \frac{2\pi}{\hbar} N_d(E_F) |\langle \psi_s | V_{scatt} | \psi_d \rangle|^2$$

$$\psi_{dm+}^1 = \textcolor{red}{a_m} \phi_m(\vec{r}) \chi_+ + \sum_n \textcolor{red}{a_{mn+}} \phi_n(\vec{r}) \chi_-$$

$$\psi_{dm-}^1 = \textcolor{red}{a_m} \phi_m(\vec{r}) \chi_- + \sum_n \textcolor{red}{a_{mn-}} \phi_n(\vec{r}) \chi_+ = \frac{1}{\sqrt{1 + \frac{3\xi^2}{2}}} \left[|x^2 - y^2 \uparrow\rangle_1 + \xi |xy \downarrow\rangle + \frac{\xi}{2} (|xz \downarrow\rangle - |yz \downarrow\rangle) \right]$$

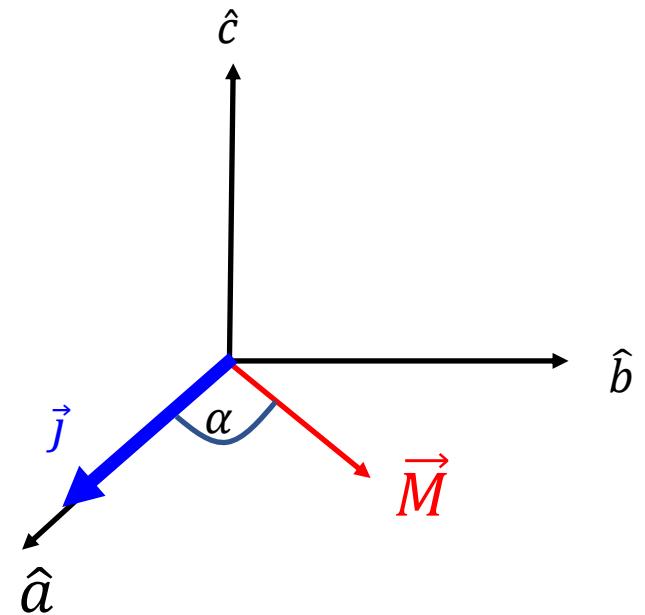


$$\sigma_{ijm++} = \frac{e^2 \hbar}{4\pi^3 m^2 v_F} \int \frac{1}{\frac{1}{\tau_{ss}} + \frac{2\pi}{\hbar} N_{dm+}(E_F) |\textcolor{red}{a_m} \langle \psi_s | V_{scatt} | \phi_m \rangle|^2} k_i k_j dS_{E_F}$$

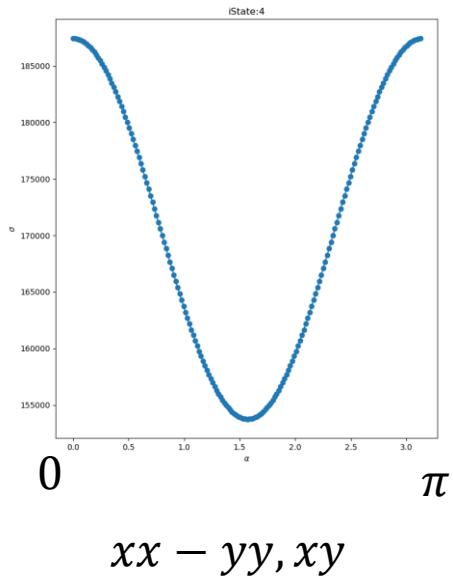
$$\sigma_{ijm+-} = \frac{e^2 \hbar}{4\pi^3 m^2 v_F} \int \frac{1}{\frac{1}{\tau_{ss}} + \frac{2\pi}{\hbar} N_{dm+}(E_F) |\langle \psi_s | V_{scatt} | \sum_n \textcolor{red}{a_{mn+}} \phi_n(\vec{r}) \chi_- \rangle|^2} k_i k_j dS_{E_F}$$

Polycrystalline AMR

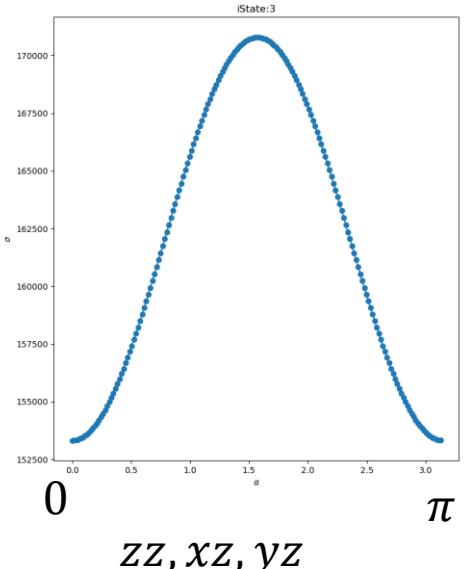
- Single-crystalline AMR depends on 4 angles.
- Polycrystalline AMR depends on only one angle between \vec{j} and \vec{M} , which needs integration over three Euler angles of the rigid body rotation.



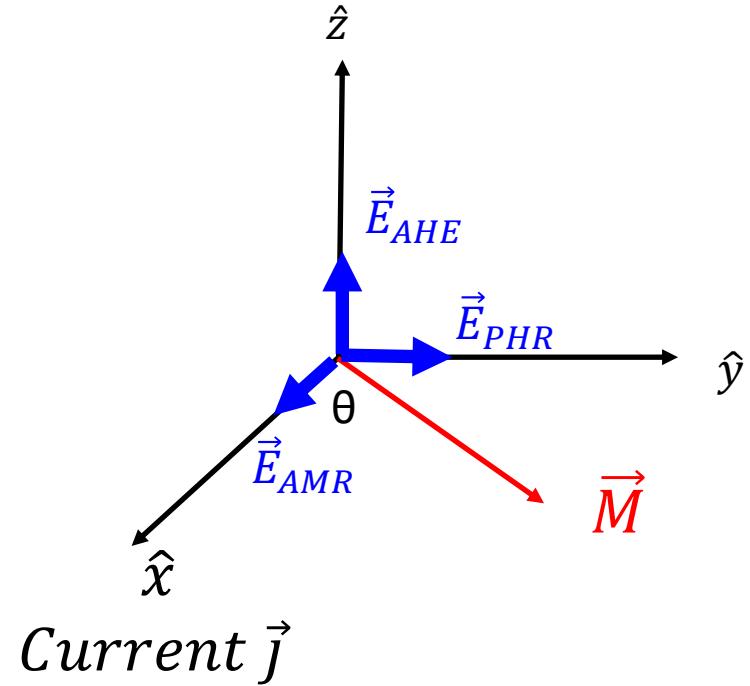
Angular dependence



$xx - yy, xy$



zz, xz, yz



Current \vec{j}

Conclusion

- Anisotropic MR is one of the angular dependence of transport caused by spin (magnetization)
- Semiclassical theory can explain AMR in terms of the relaxation time.
- Polycrystalline AMR is different on different states.