Neutron Scattering by Magnetic Crystals I: About Neutrons

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Outline

I. About Neutrons

II. General Description

II.I Neutron Diffractometer and Spectrometer

II.II Neutron Scattering Theory

II.III Applications in Condensed Matter

III. Neutron Reflectometry

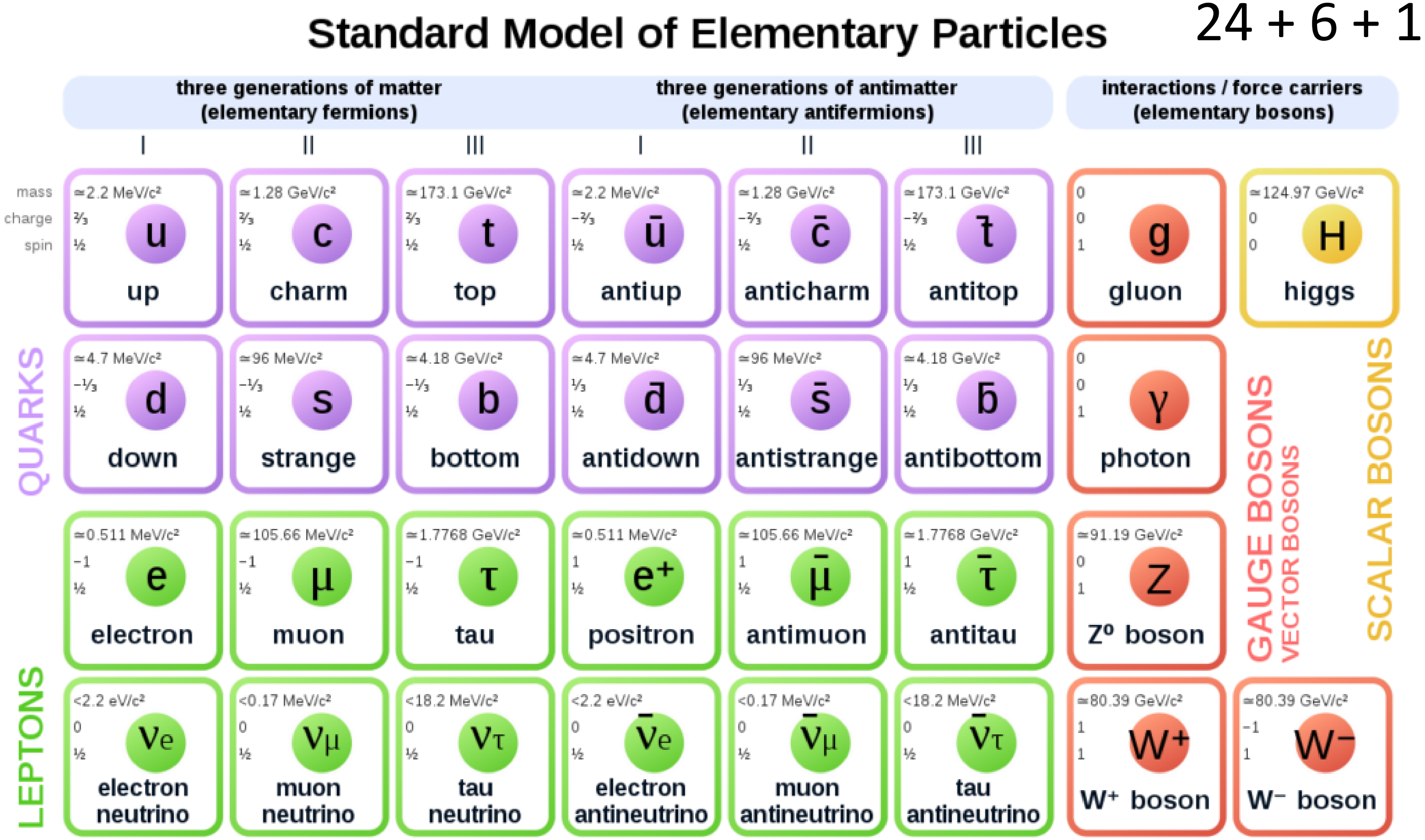
III.I Theory: specular && non-specular

III.II Neutron Reflectometer

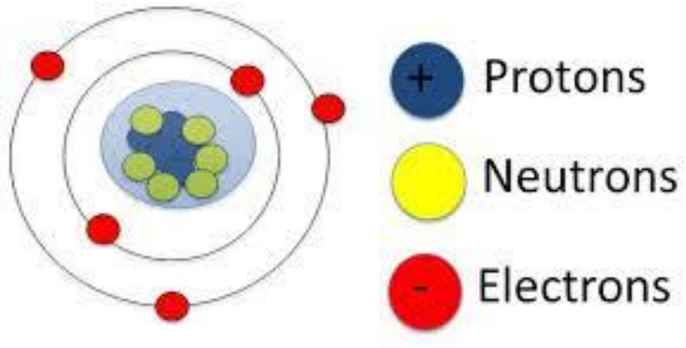
III.III Data Analysis: Model and Fitting

I. About Neutrons

Standard Model: Fundamental Particles and Interactions

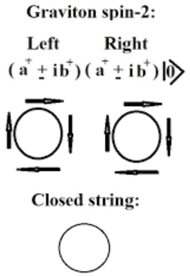


24 + 6 + 1= 31



$$SU(3)_c \times SU(2)_L \times U(1)_Y \times SL(4, \boldsymbol{R})$$

$$SU(3)_c \times U(1)_{em} \times SL(4, \boldsymbol{R})$$



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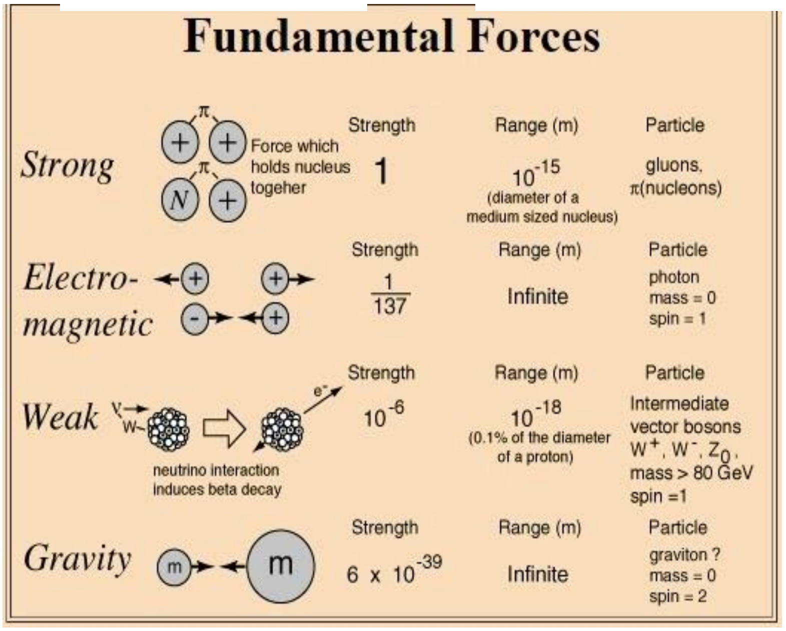
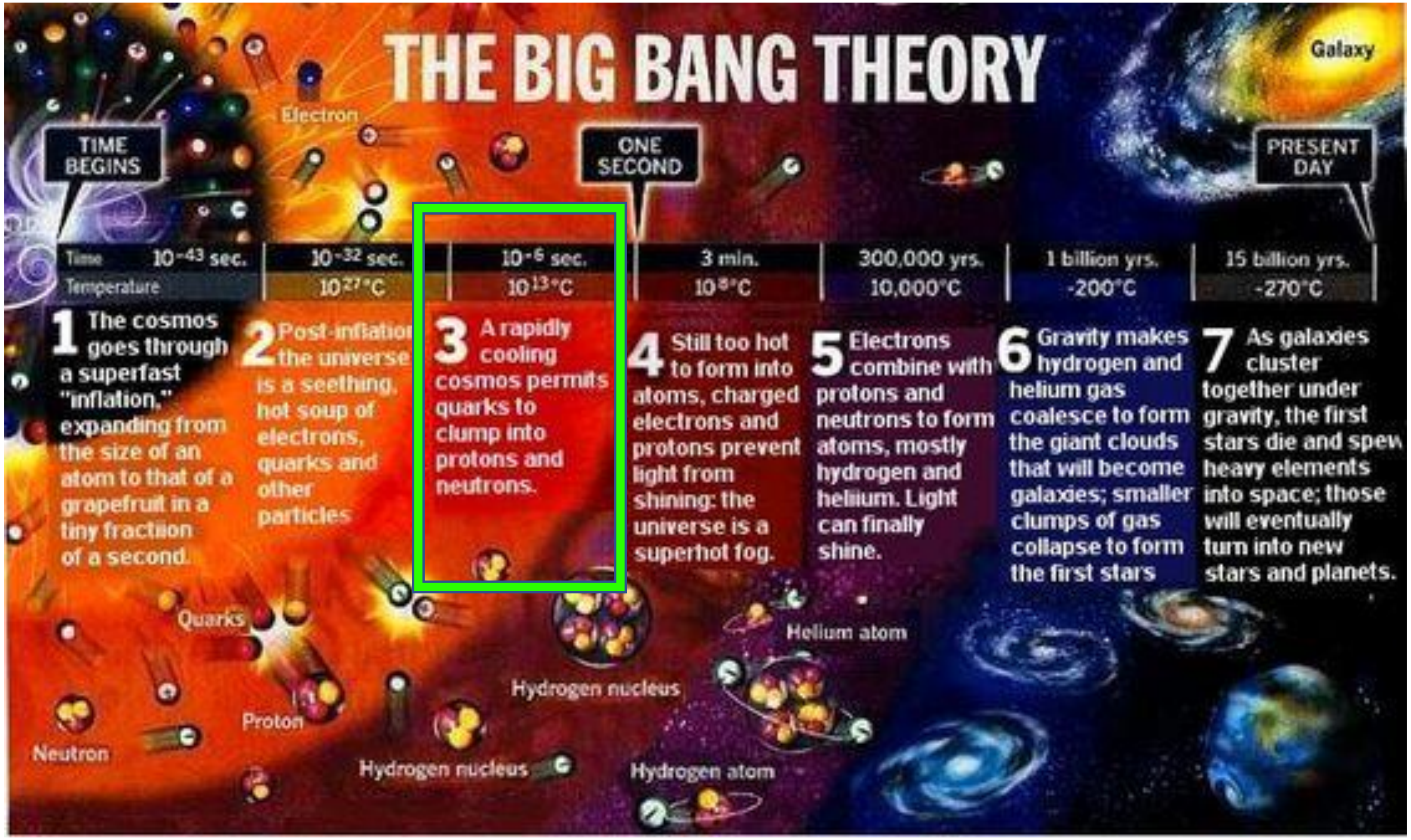
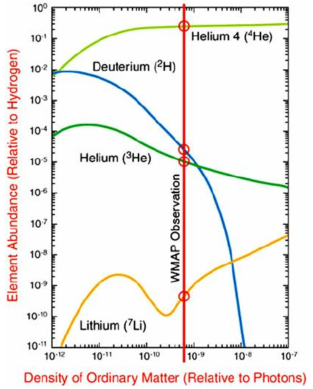


https://en.wikipedia.org/wiki/List_of_particles#Elementary_particles

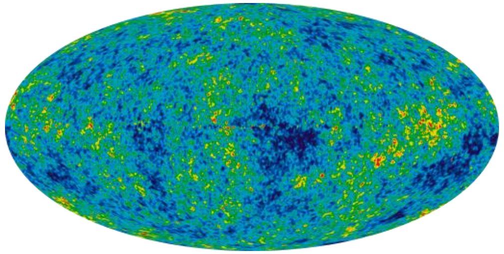
First Batch of Neutrons in the History of Universe



1) Abundance of light elements

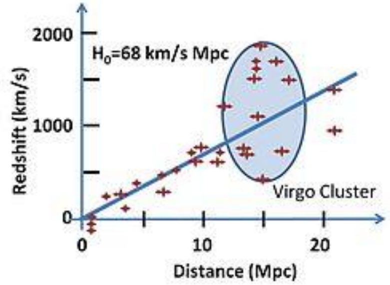


2) CMB



3)Large-scale structure

4)Hubble-Lemaitre law



String Theory: Membrane Collision

Steven Hawking : Quantum Fluctuation

Nothing → **Singularity** → **Planck Epoch** → **Grand Unification Epoch** → **Electroweak Epoch** → **Nucleosynthesis**

Singularity: $0\ s$

Planck Epoch: $g + \gamma + (Z, W) + G$; ends at $10^{-43} s$

Grand Unification Epoch: $g + \gamma + (Z, W);\ G;$ ends at $10^{-37} - 10^{-35} s$

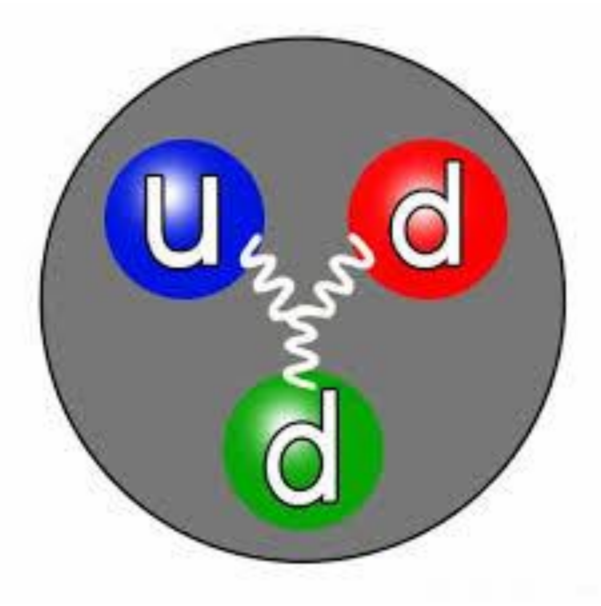
Electroweak Epoch: $g;\ \gamma + (Z, W);\ G;$ ends at $10^{-32} s$

Nucleosynthesis: Baryogenesis: birth of matter; $10^{-6} s$

Cosmic Inflation ($10^{-37} - 10^{-35} s$ to $10^{-32} s$): increased by a factor of 10^{78}

https://bigbangtheorysatit.wordpress.com/bigbang-timeline/big-bang-theory-2/

Properties of Neutron: Bond State of Quarks



$$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$$

Table 1: Neutron Properties

Gravity	Mass	m = 1.674928(1) · 10^{-27} kg
	Spin	s = -ħ/2
Electromagnetic interaction	magnetic moment	μ = -9.6491783(18)·10^{-27} JT^{-1}
Weak interaction	β-decay lifetime	τ = 885.9 ± 0.9 s
	confinement radius	R = 0.7 fm
Strong interaction	quark structure	udd

We Know Its Structure ?

$$\mathcal{L}_{QCD} = \bar{\psi}_{qi}[i(\gamma^\mu D_\mu)_{ij} - \delta_{ij} m_q]\psi_{qj} - \frac{1}{4} G^\alpha_{\mu\nu} G^{\mu\nu}_\alpha$$

$$D_\mu \equiv \partial_\mu + ig A^\alpha_\mu t_\alpha \qquad G^\alpha_{\mu\nu} \equiv \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + g f_{\alpha\beta\gamma} A^\beta_\mu A^\gamma_\nu$$

Lattice QCD →

Mass
Moments
Spin

"Spin Crisis"

We Do Not Know Its Structure !

https://www.reactor-physics.com/what-is-neutron-definition/
Neutron data booklet:https://www.ill.eu/about-the-ill/documentation/

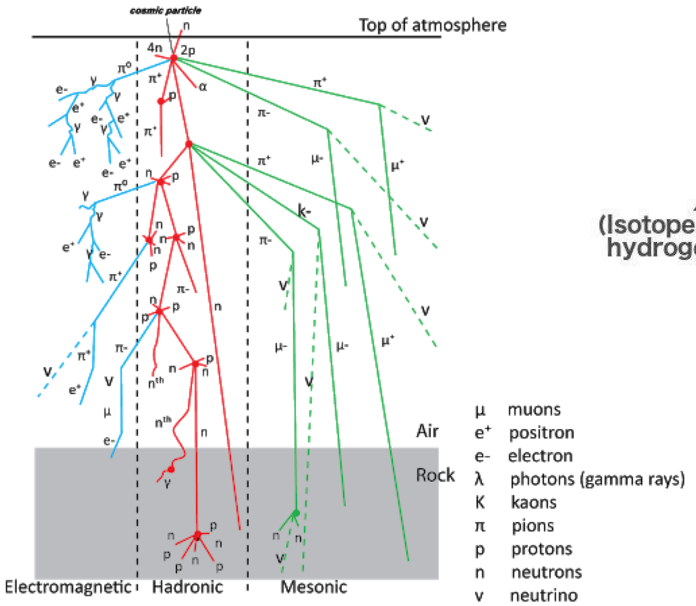
Gell, Y.; Lichtenberg, D. B. (1969). *Il Nuovo Cimento A*. Series 10. **61** (1):

Perkins, Donald H. (1982). *Introduction to High Energy Physics*. Reading, Massachusetts: Addison Wesley. pp. 201–202

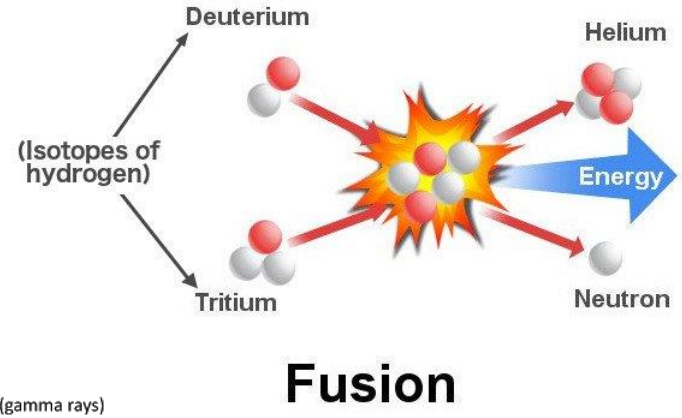
How to Get Neutrons: Sources and Mechanisms

Natural

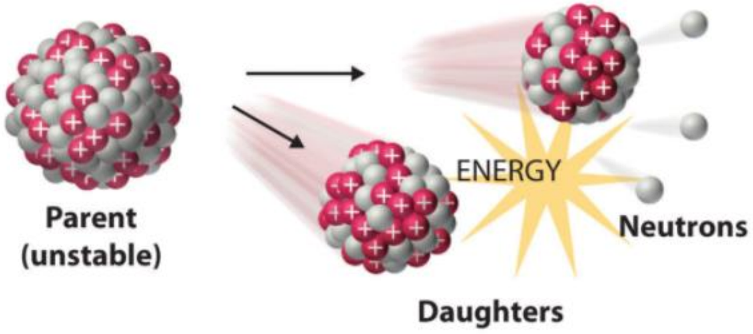
Cosmic Ray Spallation Process



Fusion Process in Stars



Spontaneous Fission Process



Artificial

(α, n)

$$\alpha + {}^{9}Be \rightarrow {}^{12}C + n$$

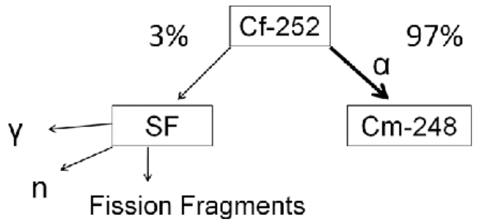
$$\alpha + {}^{9}Be \rightarrow {}^{8}Be + \alpha + n$$

$$\alpha + {}^{9}Be \rightarrow 3\alpha + n$$

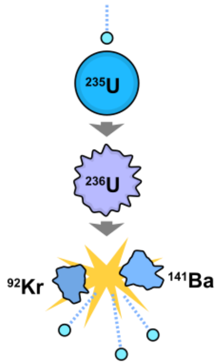
(γ, n)

$$^{9}_{4}Be + \gamma \rightarrow {}^{8}_{4}Be + {}^{1}_{0}n$$

Spontaneous Fission



Fission reactor



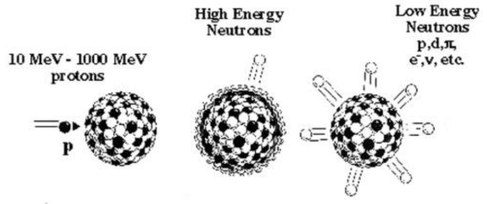
HFIR

Fusion generator

$$^{2}H + {}^{2}H \rightarrow {}^{3}He + {}^{1}n$$

$$^{2}H + {}^{3}H \rightarrow {}^{4}He + {}^{1}n$$

Spallation energetic protons



SNS

http://cosmicray.com.au/what-is-a-cosmic-ray

https://www.scienceabc.com/nature/universe/how-were-the-elements-created.html

Table of Isotopes, 8th ed., edited by R. B. Firestone and V. S. Shirley (Wiley, New York, 1996).

How to Get Neutrons: Stability of Nuclei and Elements

Strong interaction v.s. Electromagnetic Interaction

$$M\,{}_{1}^{0}p + N\,{}_{0}^{1}n \rightarrow {}_{A}^{Z}X$$

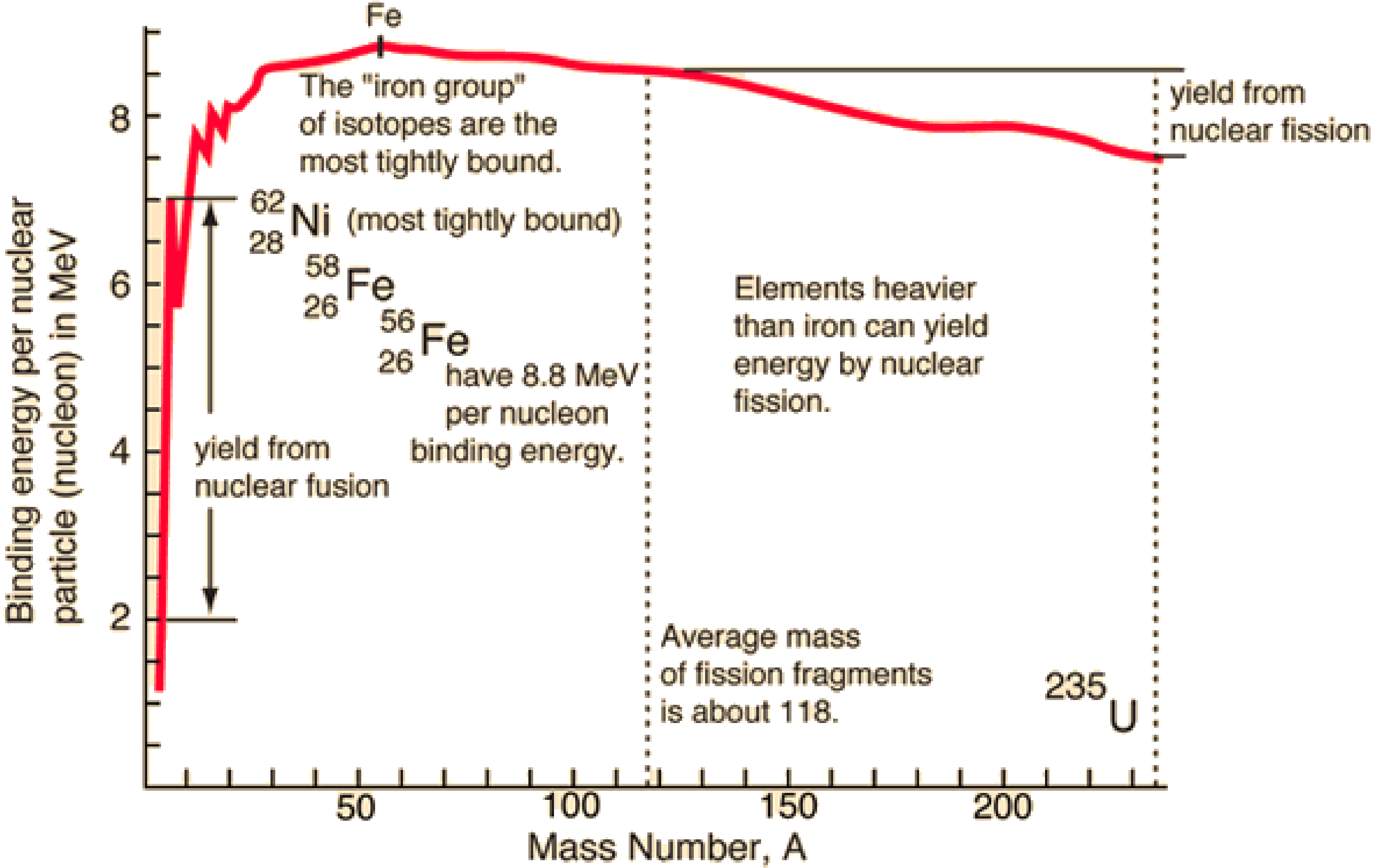
$$Z = M + N \qquad A = M$$

$$Binding\ Energy := \frac{(Mm_p + Nm_n - m_X)c^2}{M+N}$$

$${}_{Z_1}^{A_1}X + {}_{Z_2}^{A_2}Y \rightarrow {}_{Z_3}^{A_3}U + {}_{Z_4}^{A_4}V$$

Symmetry-induced selection rule:
Conversation of energy, momentum, charge, baryon number…..

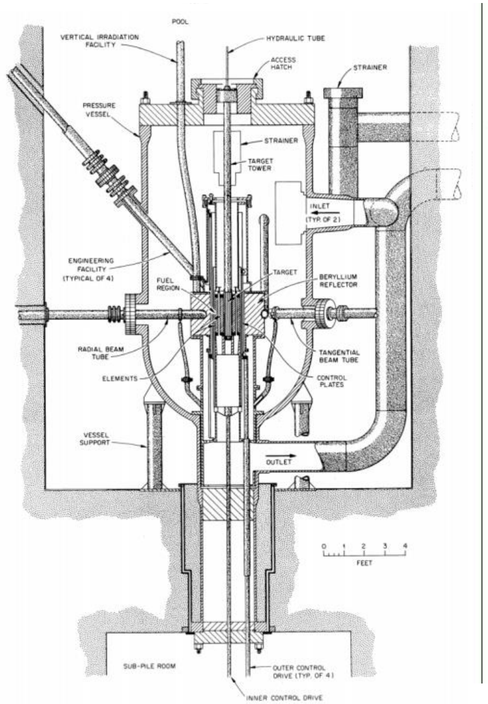
Elements: Bonded States of Nucleons

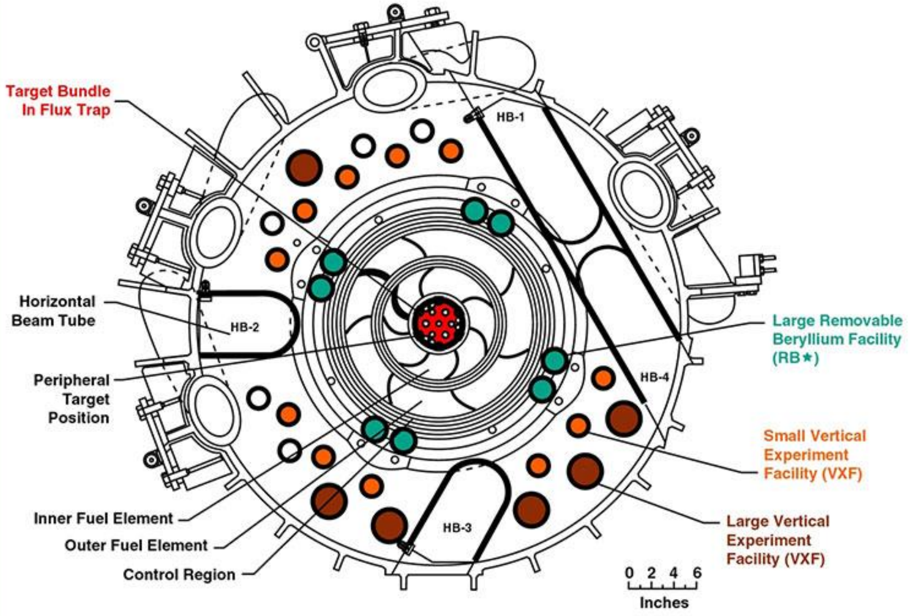


http://hyperphysics.phy-astr.gsu.edu/hbase/NucEne/nucbin.html

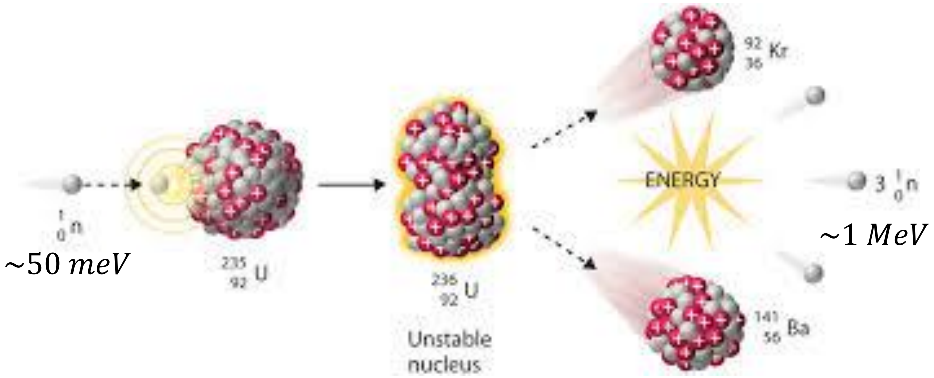
Two Neutron Sources in Oak Ridge National Lab: HFIR && SNS

THE HIGH FLUX ISOTOPE REACTOR: Highest Steady-state Thermal Flux $2.6 \times 10^5\ neutrons/(cm^2 \cdot s)$

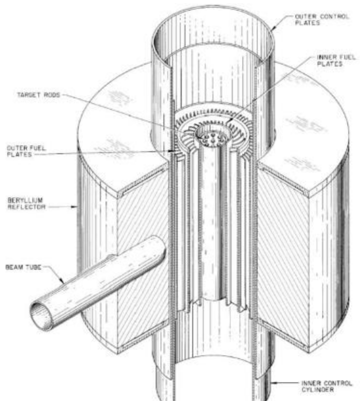


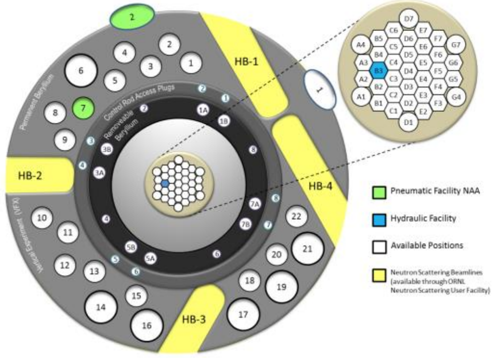


Reactor power, MW	85	
Active core height, cm	50.8	
Number of fuel elements	2	
Fuel type	U_3O_8—aluminum	
Total ^{235}U loading, kg	9.43	
Enrichment, %	93.1	
Fuel element parameters	***Inner fuel element***	***Outer fuel element***
Number of fuel plates	171	369
^{235}U loading, kg	2.60	6.83
Average fuel uranium density, gU/cm^3	0.776	1.151
^{235}U per plate, g	15.18	18.44
Burnable poison in element (^{10}B), g	2.8	None
Fuel plate thickness, cm	0.127	0.127
Coolant channel between plates, cm	0.127	0.127
Minimum aluminum clad thickness, mm	0.25	0.25
Fuel plate width, cm	8.1	7.3
Fuel cycle length, d	~24	
Cycle 400 length, d	24.6	
Coolant inlet temperature, °F	120	
Coolant outlet temperature, °F	169	
Fuel plate centerline temperature, °F	323	



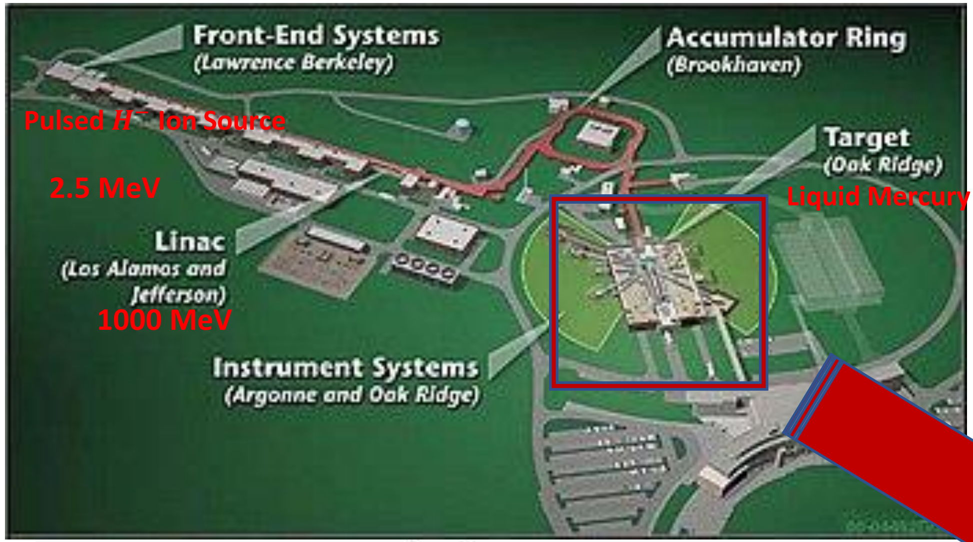
$9.4\ kg\ 235U + 2.8$ g 10B

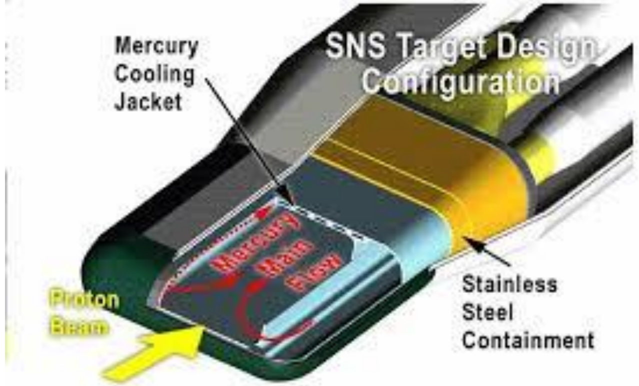


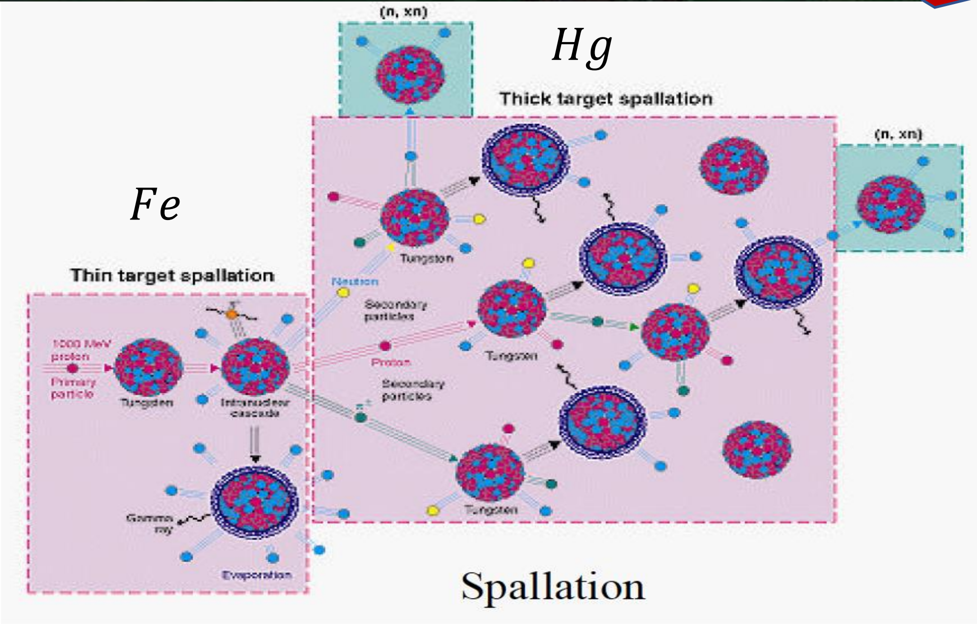


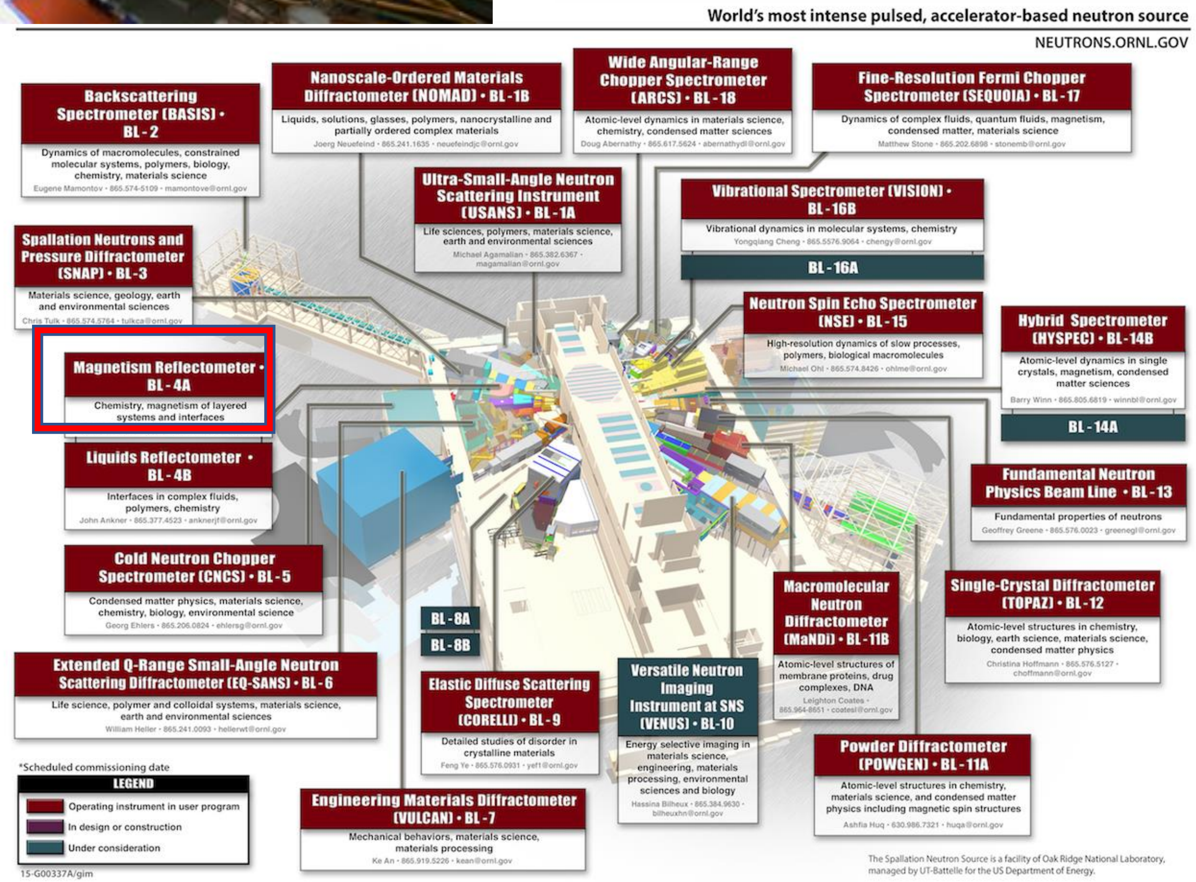
Two Neutron Sources in Oak Ridge National Lab: HFIR && SNS

Spallation Neutron Source









Neutron Scattering by Crystals

Strong interaction. && Electromagnetic Interaction

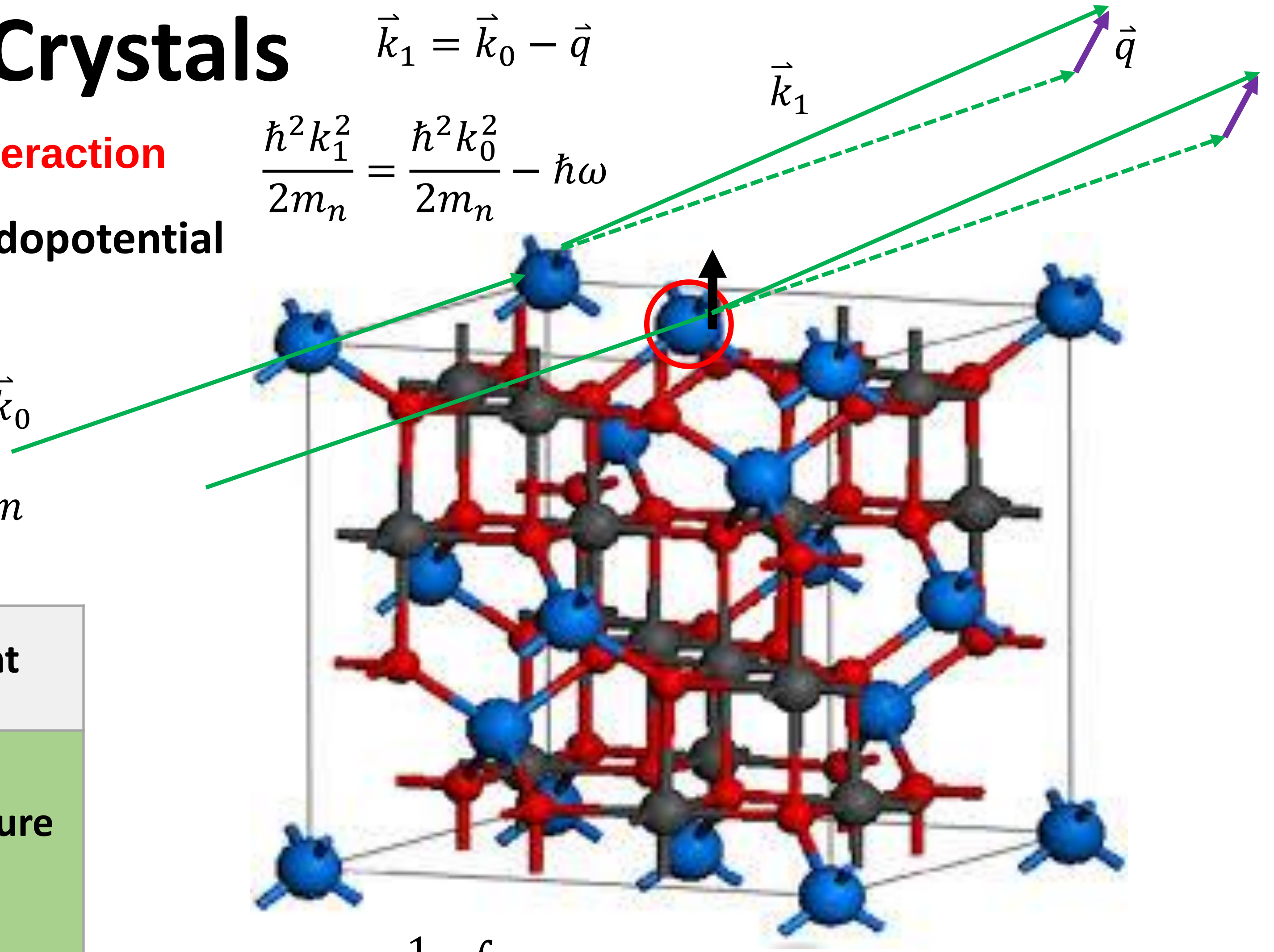
1) Neutron-Nucleus Interaction: Fermi pseudopotential

$$\boldsymbol{V(\vec{r}) = \frac{4\pi\hbar^2}{m} b_c \delta(\vec{r})}$$

$$\left(\frac{d\sigma}{d\Omega d\omega}\right)_N = \left(\frac{d\sigma}{d\Omega d\omega}\right)_{N;c} + \left(\frac{d\sigma}{d\Omega d\omega}\right)_{N;inc}$$

$$\left(\frac{d\sigma}{d\Omega d\omega}\right)_{N;\alpha} \equiv b_\alpha^2 \frac{k_1}{k_0} N S_\alpha(\vec{q},\omega),\ \alpha = c, inc$$

	Coherent	Incoherent
Elastic	Bragg diffraction: $\vec{q} = \vec{G}$	**Local structure**
Inelastic	**Single-Phonon excitation**	**NMR**



$$\vec{k}_1 = \vec{k}_0 - \vec{q}$$

$$\frac{\hbar^2 k_1^2}{2m_n} = \frac{\hbar^2 k_0^2}{2m_n} - \hbar\omega$$

$$S_\alpha(\vec{q},\omega) \equiv \frac{1}{2\pi\hbar}\int d\vec{r}dt\,G(\vec{r},t)e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

$$G(\vec{r},t) \equiv \frac{1}{N}\int d\vec{r}'\langle\rho(\vec{r}',0)\rho(\vec{r}'-\vec{r},t)\rangle \quad \rho(\vec{r}',t) \equiv \sum_j \delta(\vec{r}' - \vec{R}_j(t))$$

$$G_s(\vec{r},t) \equiv \frac{1}{N}\sum_j\int d\vec{r}'\langle\delta(\vec{r}'-\vec{R}_j(0))\delta(\vec{r}'-\vec{r}-\vec{R}_j(t))\rangle$$

Neutron Scattering by Crystals

Strong interaction. && Electromagnetic Interaction

	Coherent	Incoherent
Elastic	Bragg diffraction: $\vec{q} = \vec{G}$	**Critical Scattering**
Inelastic	**Magnon Excitation**	**Crystal Field splitting Spin-Orbital Coupling**

2) Neutron-Electron Interaction: dipole-dipole interaction

If $L = 0$, some iron group $3d$ ions

$$\left(\frac{d\sigma}{d\Omega d\omega}\right)_M = \left(\frac{d\sigma}{d\Omega d\omega}\right)_{Bragg} + \left(\frac{d\sigma}{d\Omega d\omega}\right)_{diff}$$

$$\left(\frac{d\sigma}{d\Omega d\omega}\right)_{Brag} \equiv (1.91\frac{e}{\hbar c})^2\frac{k_1}{k_0}\int\frac{dt}{2\pi}e^{-i\omega t}\langle\boldsymbol{M}_\perp(\vec{q})\rangle\langle\boldsymbol{M}_\perp(-\vec{q})\rangle$$

$$\equiv \left(1.91\frac{e}{\hbar c}\right)^2\frac{k_1}{k_0}\delta(\omega)\langle\boldsymbol{M}_\perp(\vec{q})\rangle\langle\boldsymbol{M}_\perp(-\vec{q})\rangle$$

$$\boldsymbol{M}(\vec{q}) \equiv \sum_j \mu_j e^{-i\vec{q}\cdot\vec{R}_j} \qquad \boldsymbol{M}_\perp(\vec{q}) \equiv \boldsymbol{M}(\vec{q})\cdot\vec{q}$$

$$\left(\frac{d\sigma}{d\Omega d\omega}\right)_{diff} \equiv (1.91\frac{e}{\hbar c})^2\frac{k_1}{k_0}\int\frac{dt}{2\pi}e^{-i\omega t}\left[\langle\boldsymbol{M}_\perp(\vec{q},0)\langle\boldsymbol{M}_\perp(-\vec{q},t)\rangle\rangle - \langle\boldsymbol{M}_\perp(\vec{q})\rangle\langle\boldsymbol{M}_\perp(-\vec{q})\rangle\right]$$

If $\vec{q}$ is small

$$\boldsymbol{\mu}_j = \boldsymbol{\mu}_{Lj} + \boldsymbol{\mu}_{Sj}$$

$$= \mu_B(\boldsymbol{L} + 2\boldsymbol{S})$$

If $L \neq 0$, but orbital ground state singlet or nonmagnetic doublet(some 3d ions)

$$\boldsymbol{L} + 2\boldsymbol{S} \to g\boldsymbol{S}$$

If small ion radius (rare earth ions)

$$\boldsymbol{L} + 2\boldsymbol{S} \to g\boldsymbol{J} \qquad \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S} \qquad \boldsymbol{g}\text{: Lander factor}$$