

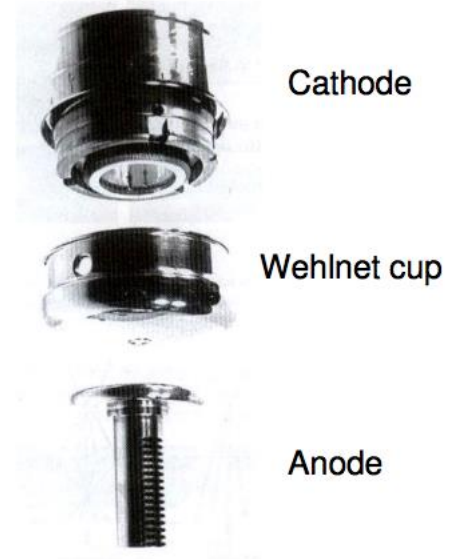
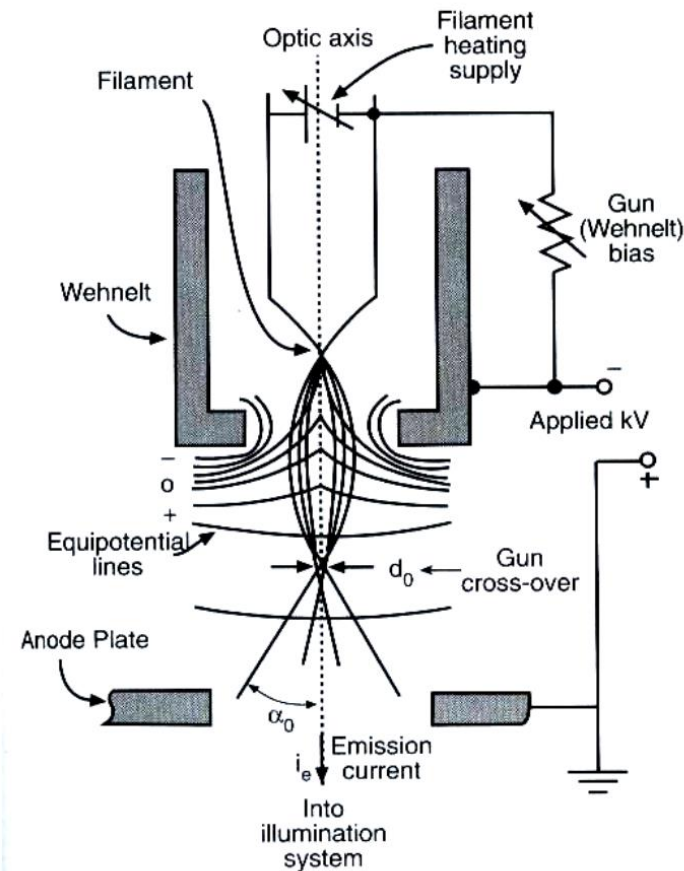
# A Simple Introduction to the Principle of Thermionic Electron Source

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# Thermionic Emission Sources

- An positive electrical potential is applied to the anode
- The filament (cathode) is heated until a stream of electrons is produced
- The electrons are then accelerated by the positive potential down the column
- A negative electrical potential (~500 V) is applied to the Whenelt Cap
- As the electrons move toward the anode any ones emitted from the filament's side are repelled by the Whenelt Cap toward the optic axis (horizontal center)
- A collection of electrons occurs in the space between the filament tip and Whenelt Cap. This collection is called a space charge
- Those electrons at the bottom of the space charge (nearest to the anode) can exit the gun area through the small (<1 mm) hole in the Whenelt Cap
- These electrons then move down the column to be later used in imaging



Thermionic Gun

# Richardson's Law

The emitting current density  $J(\text{A}/\text{m}^2)$  is related by the temperature  $T$  by:

1901: 
$$J = A_G T^2 e^{-\frac{\Phi}{kT}}$$

1911-1930: 
$$A_G = \lambda_R A_0$$

$\lambda_R \sim 0.5$

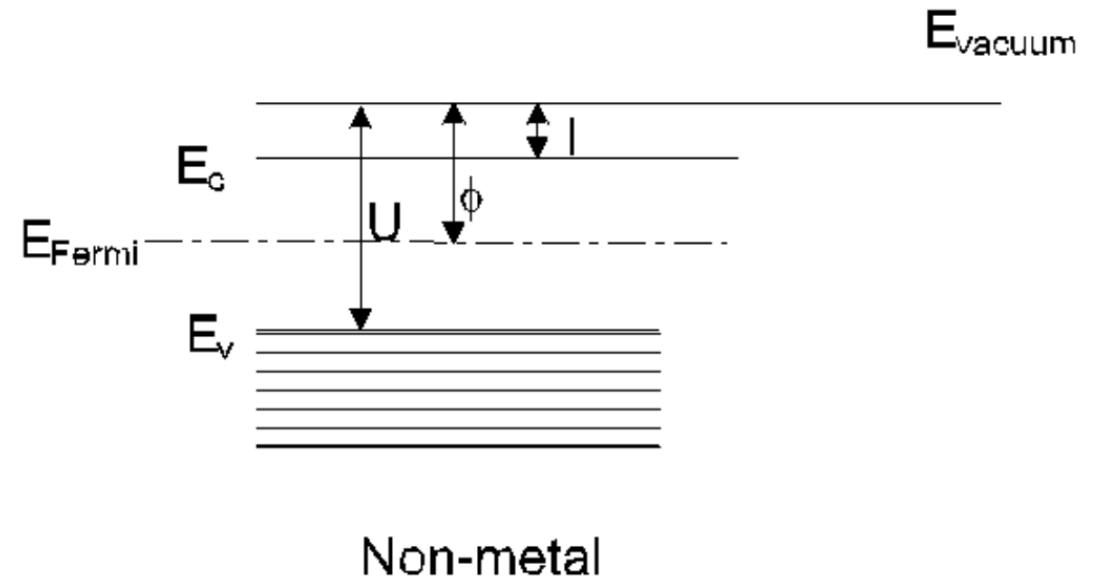
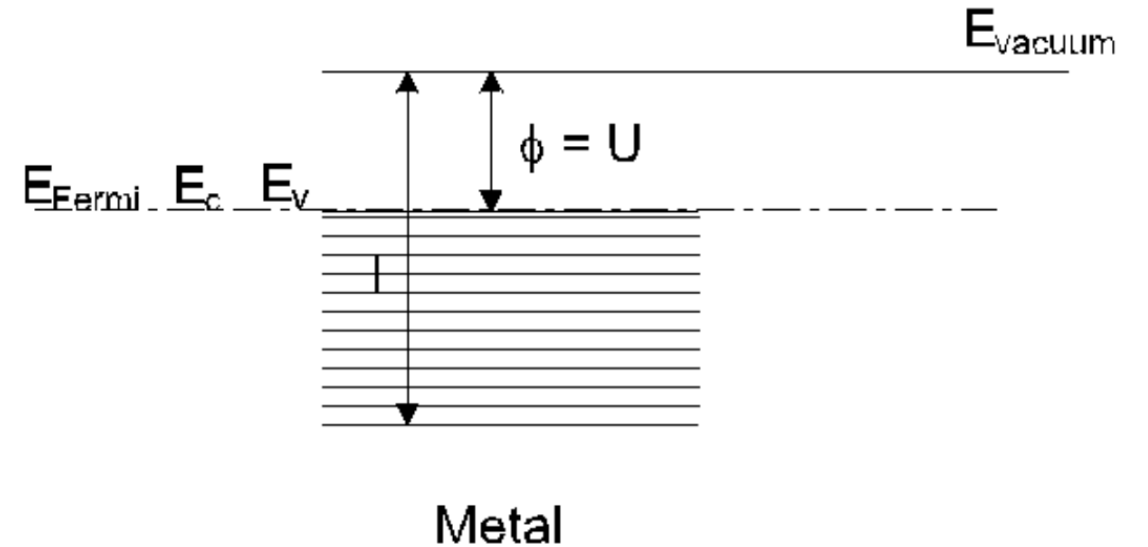
$$A_0 = \frac{4\pi m k^2 q_e}{h^3} = 1.20173 \times 10^6 \text{ A m}^{-2} \text{ K}^{-2}$$

1930- : 
$$\lambda_R = \lambda_B (1 - r)$$

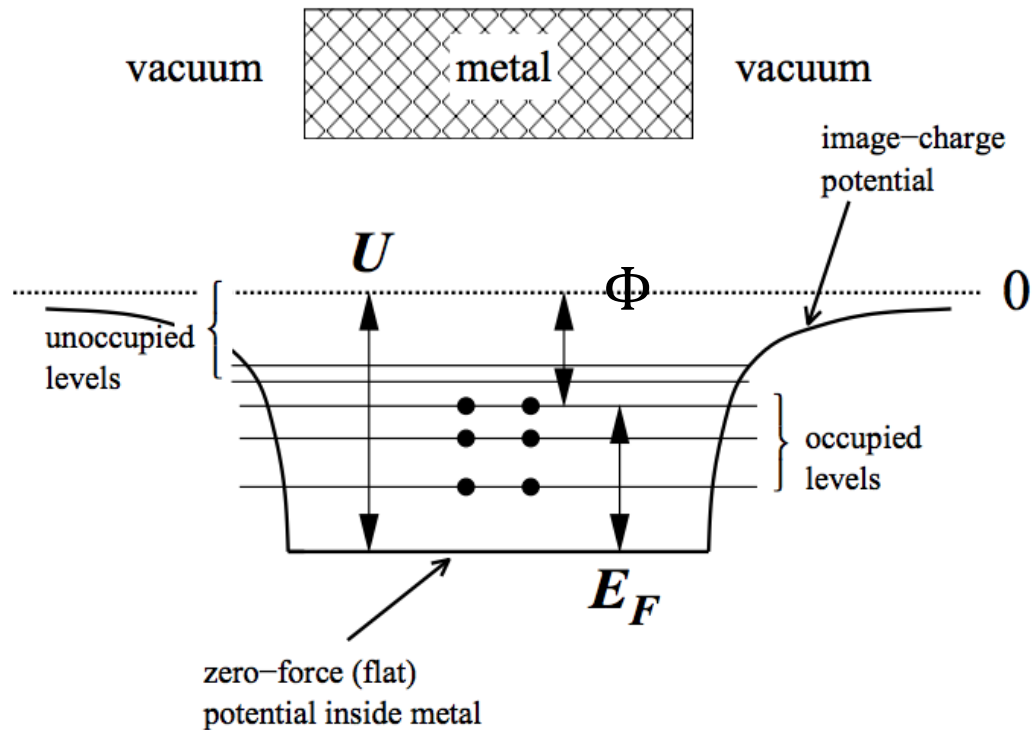
$\lambda_R$  vary with different materials and different crystal surfaces of the same material

**Work Function:** the minimum thermodynamic work needed to remove an electron from a solid to a point in the vacuum immediately outside the solid surface.

- 1) Here "immediately" means that the final electron position is far from the surface on the atomic scale, but still too close to the solid to be influenced by ambient electric fields in the vacuum.
- 2) The work function is not a characteristic of a bulk material, but rather a property of the surface of the material (depending on crystal face and contamination)



# *A simple semi-classical derivation of Richardson's Law*



$$\Phi = U - E_F$$

1, Free electron gas

2, Fermi-Dirac distribution

# Free electron gas

$$H \Psi_n(\mathbf{r}) = \frac{p^2}{2m} \Psi_n(\mathbf{r}) = -\frac{\hbar^2}{2m} \nabla^2 \Psi_n(\mathbf{r}) = \varepsilon_n \Psi_n(\mathbf{r})$$

Periodic boundary condition:  $\Psi_n(x + L, y, z) = \Psi_n(x, y, z)$

- **Traveling plane wave solution:**

$$\Psi_{\mathbf{k}}(\mathbf{r}) = A \exp(i\mathbf{k} \cdot \mathbf{r})$$

$$k_x = \frac{2\pi n_x}{L}, \quad k_y = \frac{2\pi n_y}{L}, \quad k_z = \frac{2\pi n_z}{L}$$

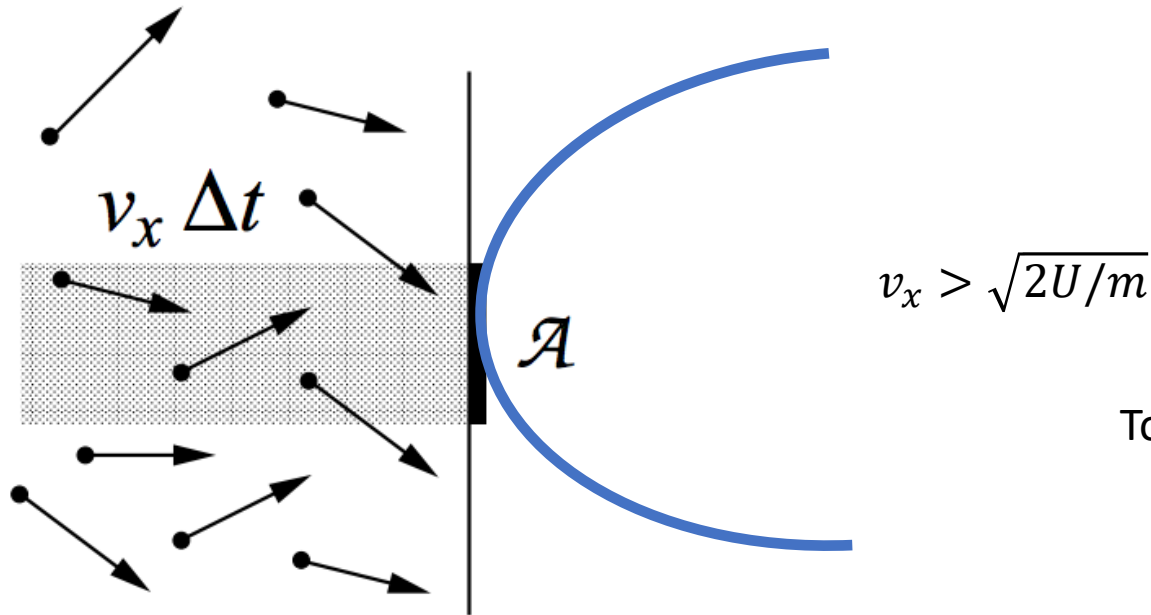
$$\varepsilon_{\mathbf{k}} = \frac{\hbar^2}{2m} (k_x^2 + k_y^2 + k_z^2)$$

$$\frac{\hbar \mathbf{k}}{m} = \mathbf{v} \quad v_i = \frac{2\pi \hbar n_i}{mL}, \quad i = x, y, z$$

$$dv_i = \frac{2\pi \hbar dn_i}{mL}, \quad i = x, y, z$$

$$dv_x dv_y dv_z = (2\pi \hbar / mL)^3 dn_x dn_y dn_z$$

$$dn_x dn_y dn_z / L^3 = (2\pi \hbar / m)^{-3} dv_x dv_y dv_z$$



In order for an electron to escape during some time  $\Delta t$ , it must 1) have  $v_x$  sufficient to overcome the image-charge barrier  $U$  ; 2) it must be sufficiently close to the wall.

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

$$dn_x dn_y dn_z / L^3 = (2\pi\hbar/m)^{-3} dv_x dv_y dv_z$$

$$E = \frac{1}{2} m v^2$$

Total electron number that can escape:

$$N = \int_{\sqrt{2U/m}}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z 2 \left(\frac{2\pi\hbar}{m}\right)^{-3} f(E) A v_x \Delta t$$

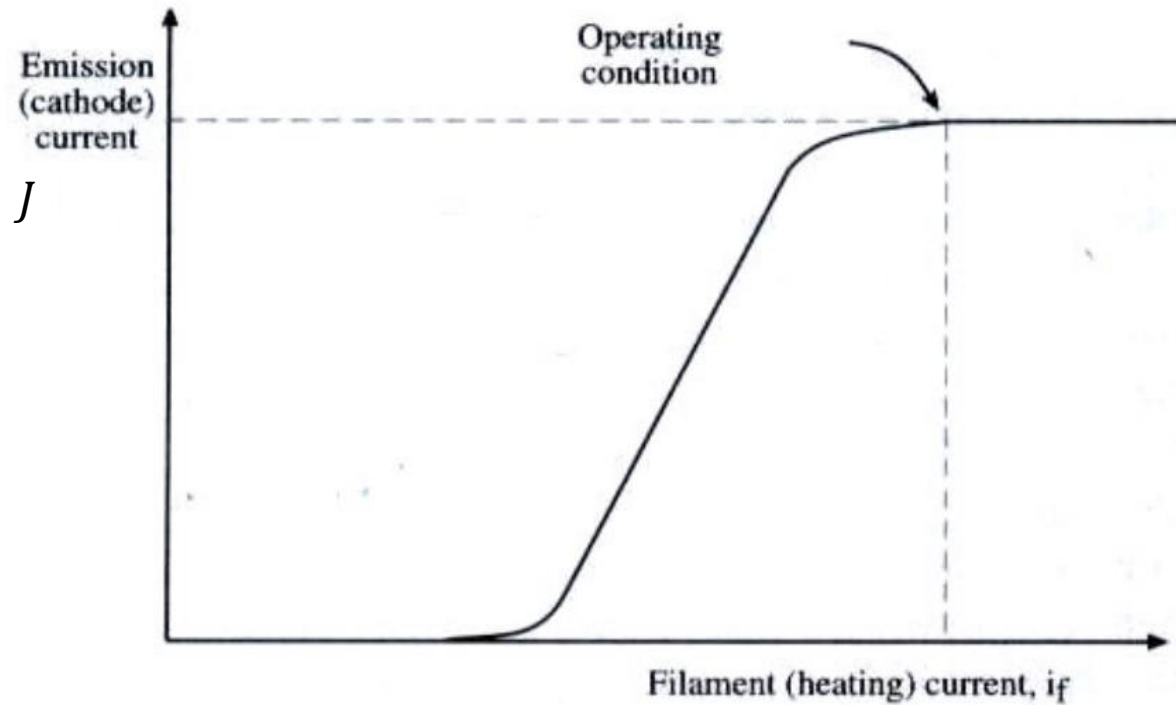
$$J = \frac{eN}{A\Delta t} = \int_{\sqrt{2U/m}}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z 2 \left(\frac{2\pi\hbar}{m}\right)^{-3} f(E) v_x$$

if  $E \gg E_F$        $f(E) \approx \exp\left(-\frac{E - E_F}{kT}\right)$

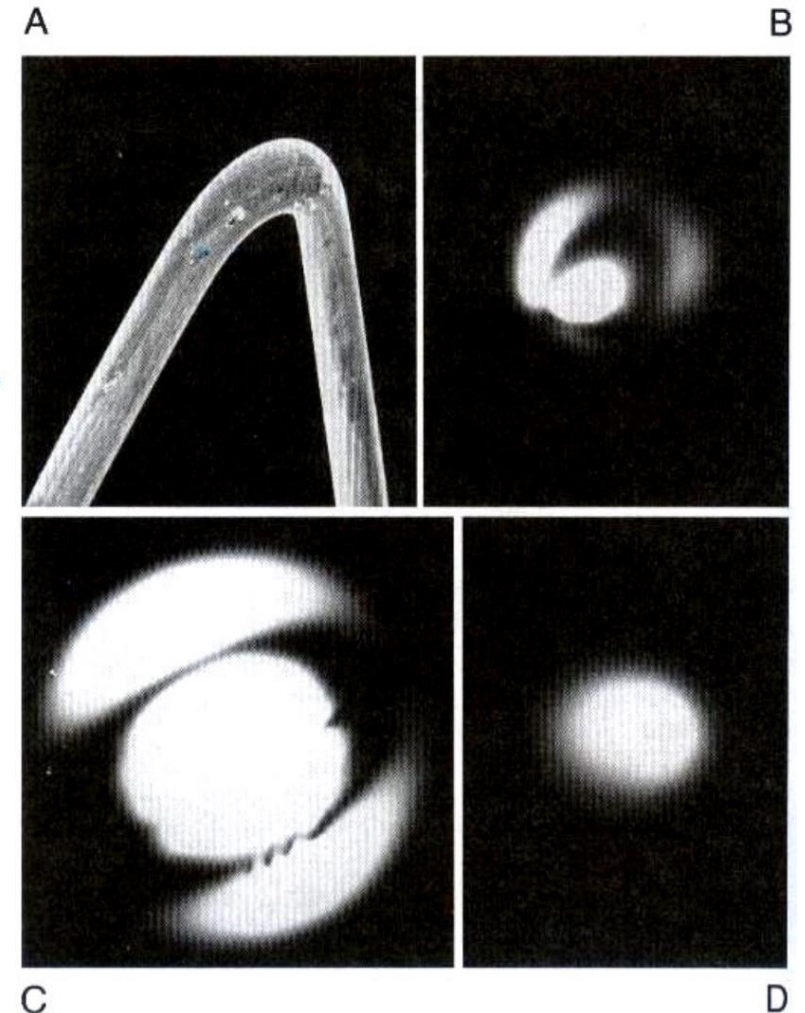
$$J = \frac{4\pi e m (kT)^2}{h^3} e^{-\frac{\Phi}{kT}}$$

$$J = A_G T^2 e^{-\frac{\Phi}{kT}}$$

# Saturation Condition



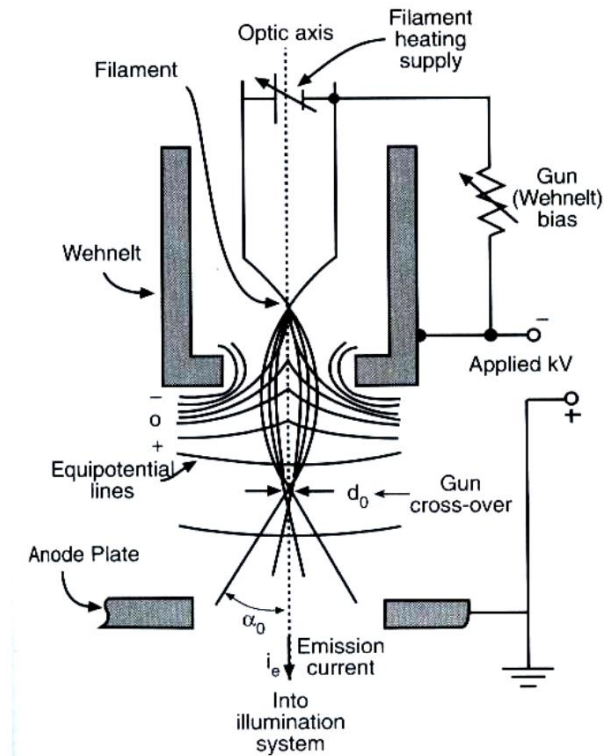
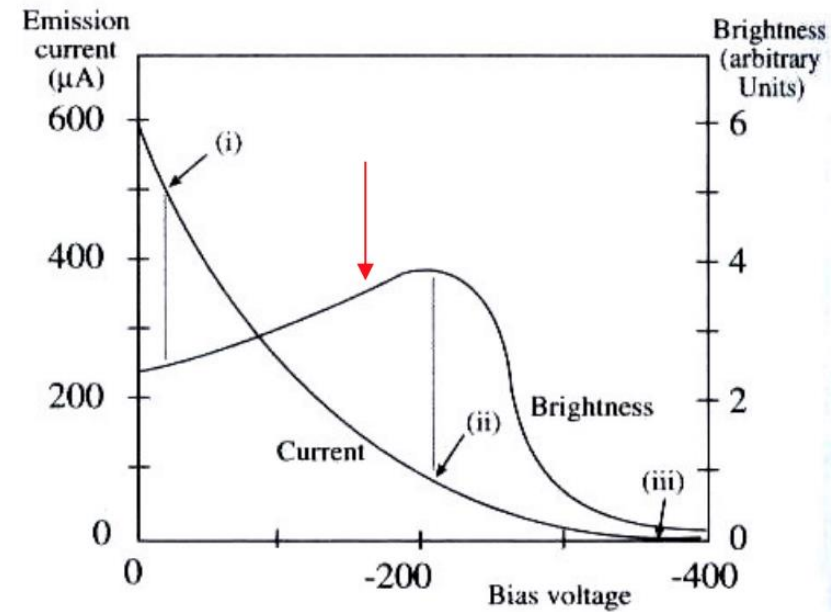
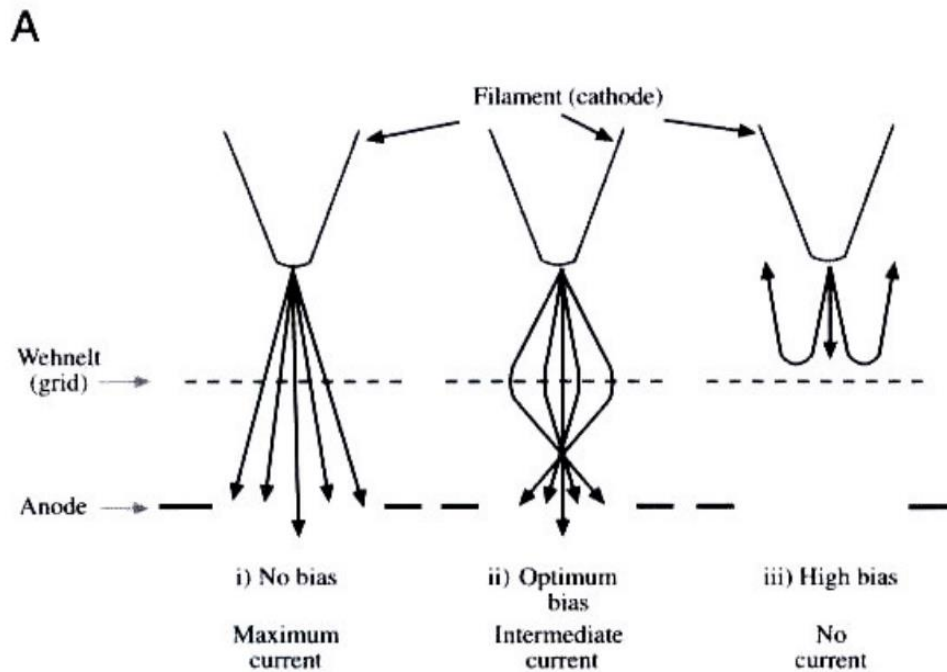
High temperature heating give higher  $J$  but shorten the source life through evaporation/ oxidation  
 Less than saturation decreases the intensity of the signals  
 Higher than saturation decreases the life of filament



**Figure 5.5.** (A) The tip of a tungsten hairpin filament and the distribution of electrons when the filament is (B) undersaturated and misaligned, (C) undersaturated and aligned, and (D) saturated.



# Optimize the Beam Current



**Figure 5.4.** (A) The effect of increasing Wehnelt bias (i–iii) on the distribution of electrons coming through the anode. (B) The relationship between the bias and the emission current/gun brightness. Maximum brightness is achieved at an intermediate Wehnelt bias, and an intermediate emission current [condition (ii) in A].

## Thermionic Gun

SEM, small probe, no Wehnelt bias

TEM, brighter image, Wehnelt bias control needed

# Characteristics of Electron Beam

## Brightness

Current density per unit solid angle

$$\beta = \frac{i_e}{\pi \left(\frac{d_0}{2}\right)^2 \pi (\alpha_0)^2} = \frac{4 i_e}{(\pi d_0 \alpha_0)^2}$$

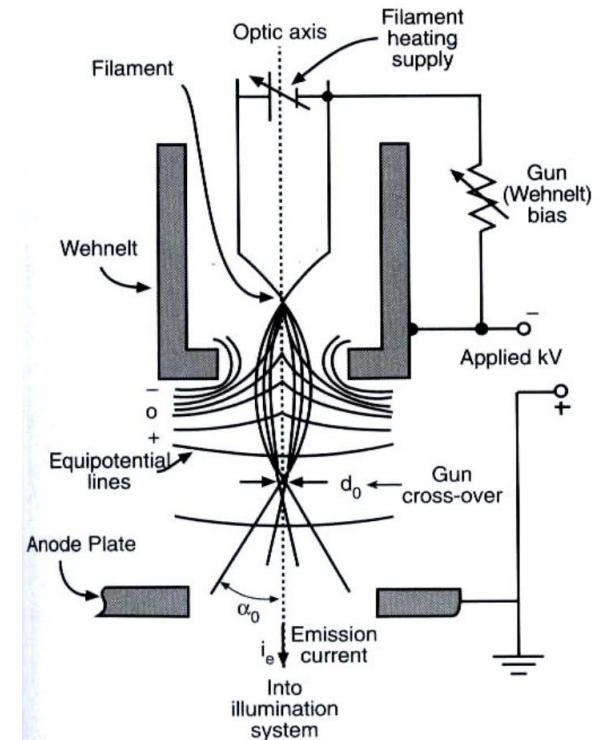
Units of  $\beta$  is  $\text{A.cm}^{-2}\text{sr}^{-1}$

More is  $\beta$ , more is no of electrons/area

More beam damage

Important with fine beams, as in AEM

TEM uses defocused beam



**Figure 5.1.** Schematic diagram of a thermionic electron gun. A high voltage is placed between the filament and the anode, modified by a potential on the Wehnelt which acts to focus the electrons into a crossover, with diameter  $d_0$  and convergence/divergence angle  $\alpha_0$ .

# Characteristics of Electron Beam

Coherence length

$$\lambda_c = \frac{vh}{\Delta E}$$

where  $h$  is Planck's constant,  $v$  is velocity of the electrons and  $\Delta E$  is the energy spread of the beam

$\Delta E$  related to stability of accelerating voltage

Typical  $\Delta E$  values are 0.1 – 3eV. Electron energies are up to 400keV

Not much important for imaging

Important in spectroscopy, EELS

$\Delta E$  measured using an electron spectrometer

$\Delta E$  is taken as the FWHM of the Gaussian peak obtained

# Spatial Coherency

Related to the size of the source

Perfect source – electron emanating from same point

Effective source size for coherent illumination

where  $\lambda$  is the Wavelength and  
 $\alpha$  is angle subtended by source  
at specimen

$$d_c \ll \frac{\lambda}{2\alpha}$$

$d_c$  should be as large as possible

$\alpha$  is limited by source size or aperture size

Small beams are more spatially coherent

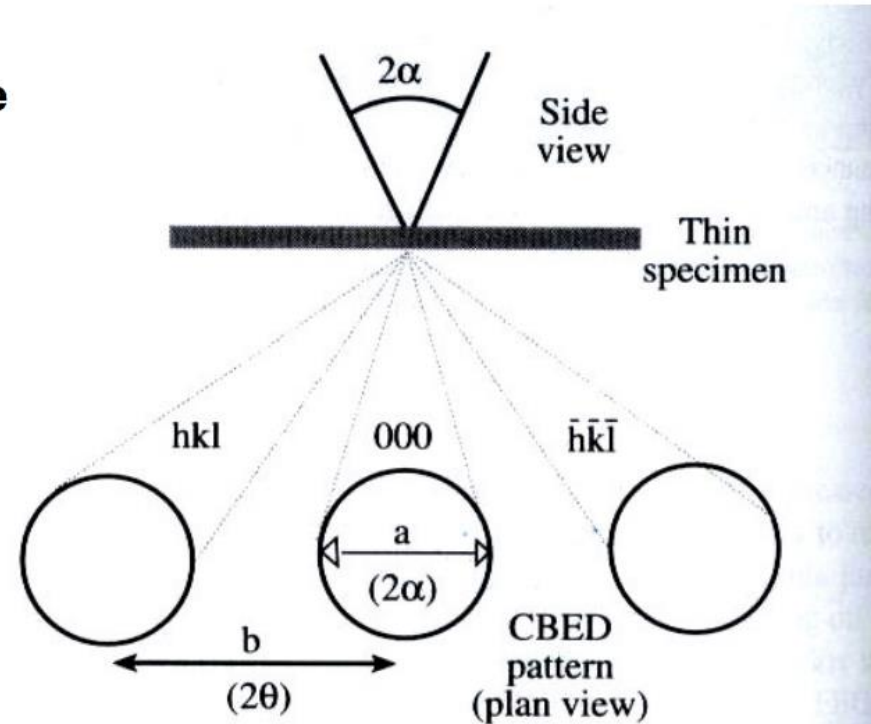
Required for good phase contrast and  
diffraction patterns

Convergence Angle Determination

$$2\alpha = 2\theta_B \frac{a}{b}$$

$\alpha$  Important in Brightness calculation,  
CBED, STEM and EELS

$\alpha$  controlled by final aperture



**Figure 5.8.** The distances on a convergent-beam diffraction pattern from which you can measure the beam-convergence semiangle,  $\alpha$ , which is proportional to the width of the diffraction disk.

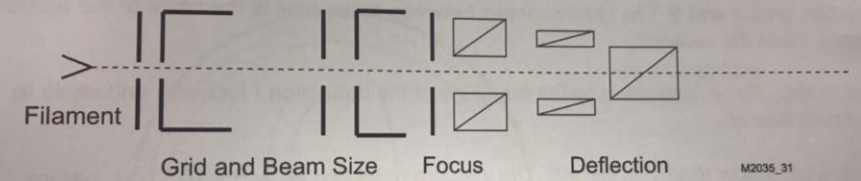
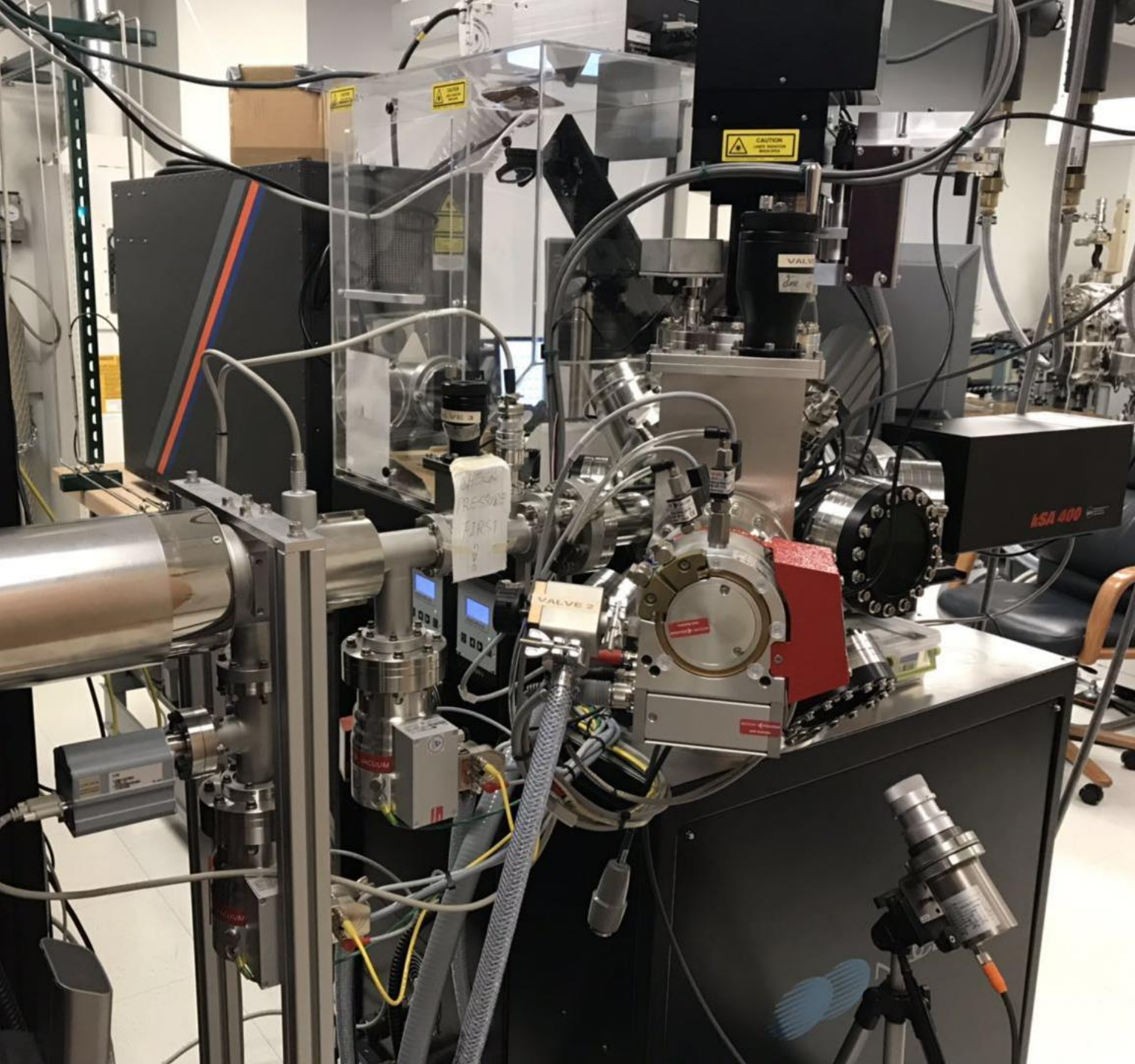


Figure 3.1 Principle of Source Operation

### 3.2 Beam Properties

The source produces a beam with the following characteristics:

	Minimum	Maximum
Beam energy	1 keV	35 (30) keV
Beam intensity	0.01 $\mu\text{A}$	140 (120) $\mu\text{A}$
Focus position**	100 mm	1 m
Focus size*	40 $\mu\text{m}$ (50 $\mu\text{m}$ )	5 mm
Divergence*	$1.10^{-4}$ rad	-

\* See notes concerning the influence of AC-magnetic fields

\*\* Measured from the mounting flange

The smallest attainable focus spot lies between 40  $\mu\text{m}$  and 5 mm, depending on the working distance. To observe and find the best focus, set the beam intensity below

# Our System

