Arrott and Arrott-Noakes Magnetism Plots

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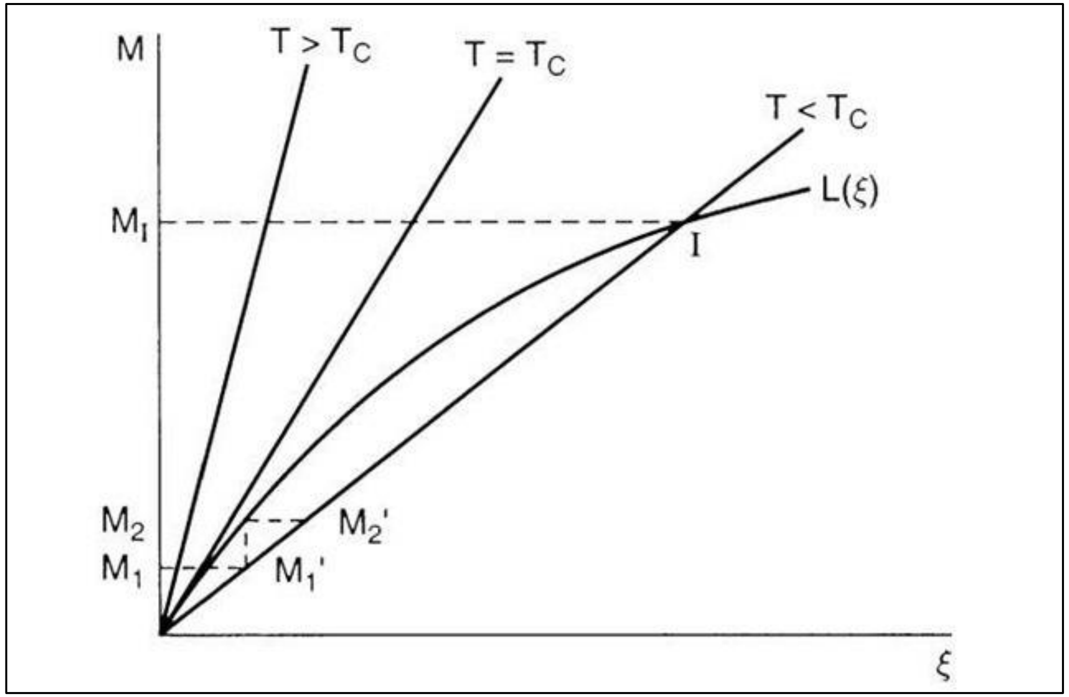
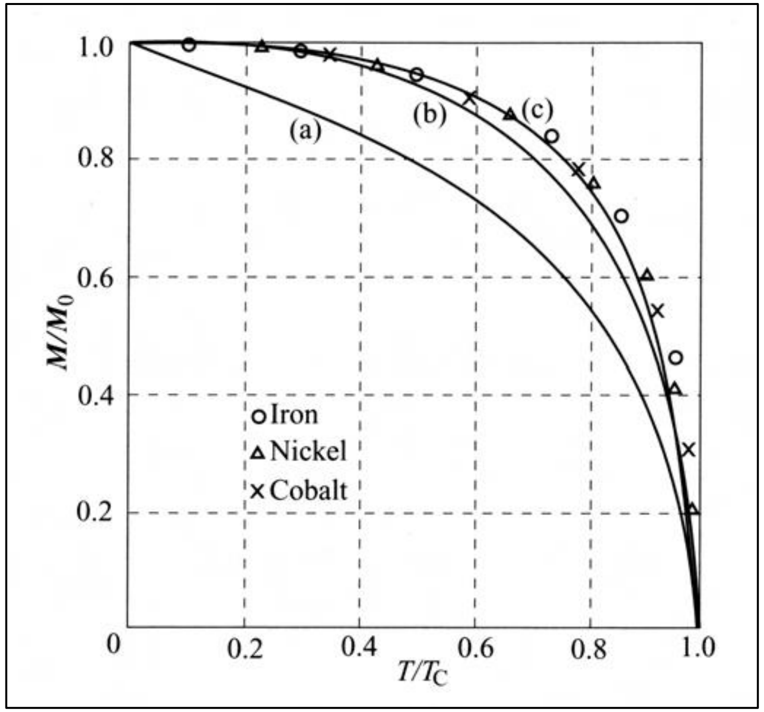
Classical PM-to-FM Ordering

- Magnetization follows the *Langevin function*
  $$L(x) \equiv ctanh(x) - \frac{1}{(x)}; x = \frac{\mu H}{k_B T}$$
  $\approx \frac{1}{3}x - \frac{1}{45}x^3$ for $x \ll 1$.
- Weiss Mean Field Theory: magnetic moments experience magnetic field proportional to the *mean field* of the nearby atoms H $\rightarrow H + \lambda m$
  - $m = N\mu L\left(\frac{\mu(H+\lambda m)}{k_B T}\right)$; expand to 1st order in x
- $m \approx \frac{N\mu}{3}\frac{\mu(H+\lambda m)}{k_B T} \Rightarrow \frac{C}{T}(H + \lambda m) = m, \frac{m}{H} = \chi = \frac{C}{T-\lambda C} = \frac{C}{T-T_c}$
- In other words, Weiss molecular field λ brings about susceptibility divergence at nonzero transition temperature

PM-to-FM Ordering

- For general temperature, have to solve $m = N\mu L\left(\frac{\mu(H+\lambda m)}{k_B T}\right)$
- Graphical method: plot line m, Langevin function and find intercept
- Spontaneous magnetization as function of temperature: shape matches what's found experimentally





How to Determine T_c?

- $\chi = \frac{C}{T - T_c}$, so can't we just measure χ and look for where it diverges?
  - Ok for getting decent idea of T_c, but χ never actually "diverges" in practice
  - If need more precise T_c, need another method

- Revisit Weiss mean field expression: $m = N\mu L(x) \approx \frac{N\mu}{3}x - \frac{N\mu}{45}x^3$. Using $x = \frac{\mu}{k_B T}(H + \lambda m)$:
- $m = \frac{A\mu}{k_B T}H + \frac{A\mu\lambda}{k_B T}m - \frac{B\mu^3\lambda^3}{(k_B T)^3}m^3$, or
- $\frac{A\mu H}{k_B T} = \frac{B\mu^3\lambda^3}{(k_B T)^3} - \left(\frac{T_c}{T} - 1\right)m$, square and solve for m:
- $m^2 = \frac{(k_B T)^3}{B\mu^3\lambda^3}(T_c - T) + \frac{A}{B}\frac{(k_B T)^2}{\mu^2\lambda^3}\frac{H}{m}$

Plot m^2 vs H/m for temperatures around T_c, look for which temp intersects the origin

Arrott Plots

<2K difference

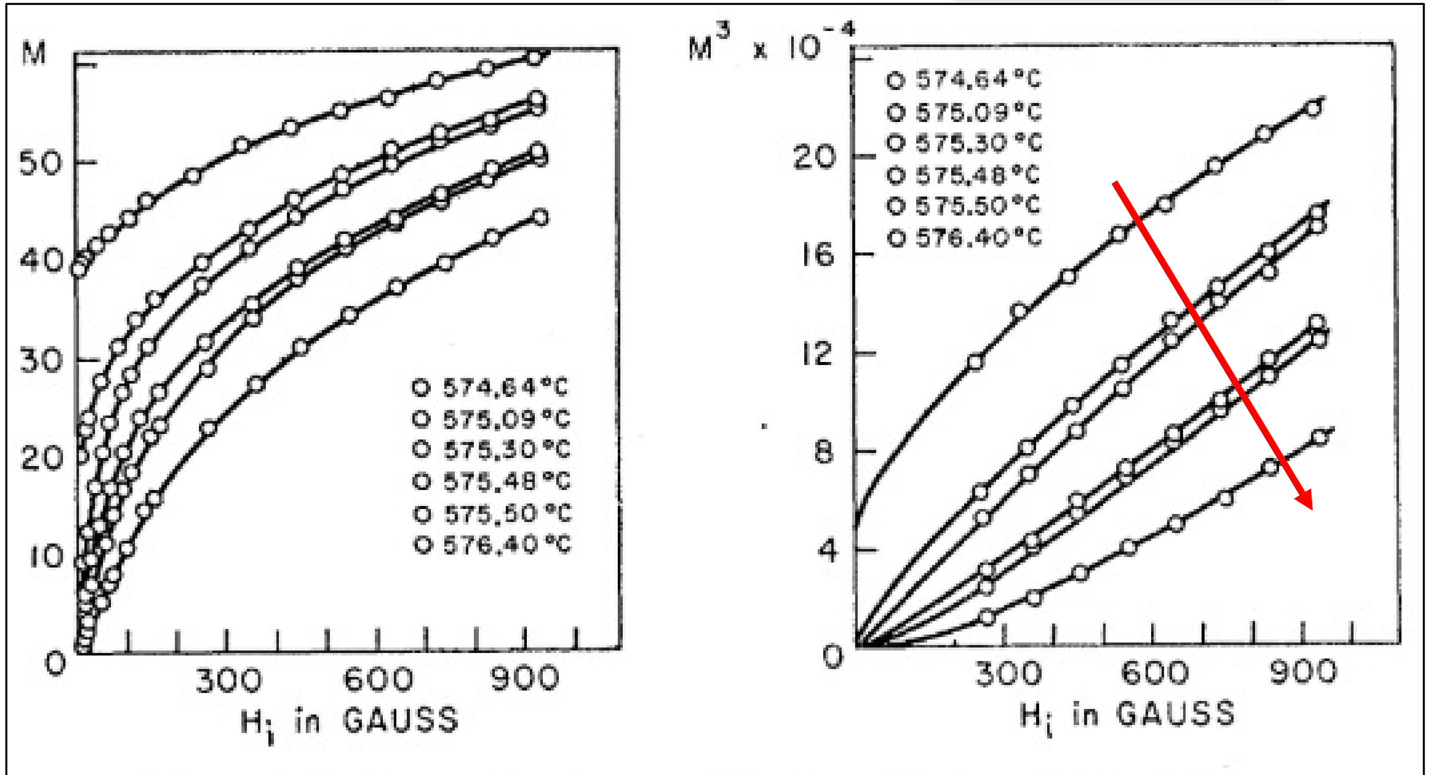
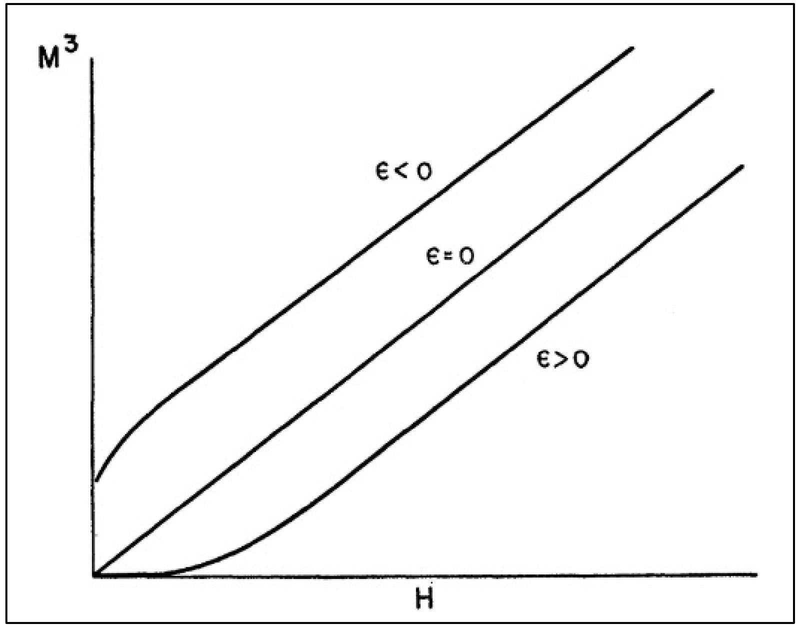


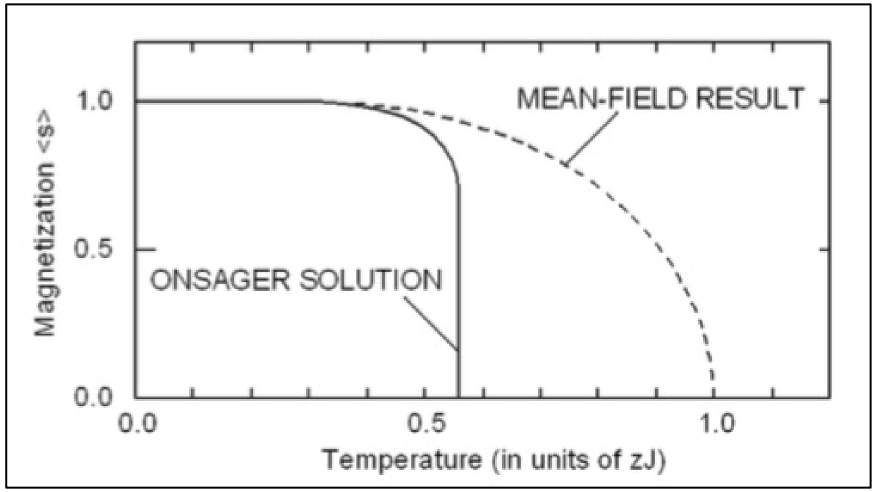
FIG. 3. Magnetization of Fe_3O_4 (data of Smith[1]).



A. Arrott, Phys. Rev. **108**, 1394 (1957)

Deviations from Mean Field

- Near T_c, fluctuations are great enough to invalidate Weiss' mean field approximation.
- Depend on *dimensionality of system* (e.g. 1d chain, 2d lattice, etc)
  - 1d: T_c can only be zero (Ising solved exactly)
  - 2d: $T_c>0$ can exist (Onsager; 1968 Nobel Prize)
  - 3d: correction to MFT required
- *Spin dimensionality* important as well
  - 1d spins: Ising model
  - 2d: XY model
  - 3d: Heisenberg model



Arrott-Noakes Plots

- More general equations of state:
- $M^{\frac{1}{\beta}} = \frac{T-T_c}{T_1} + \left(\frac{H}{M}\right)^{\frac{1}{\gamma}}$
  - β=0.5, γ=1 in mean field
  - Intersect origin @ T=T_c; linearize plots with appropriate choices of exponents

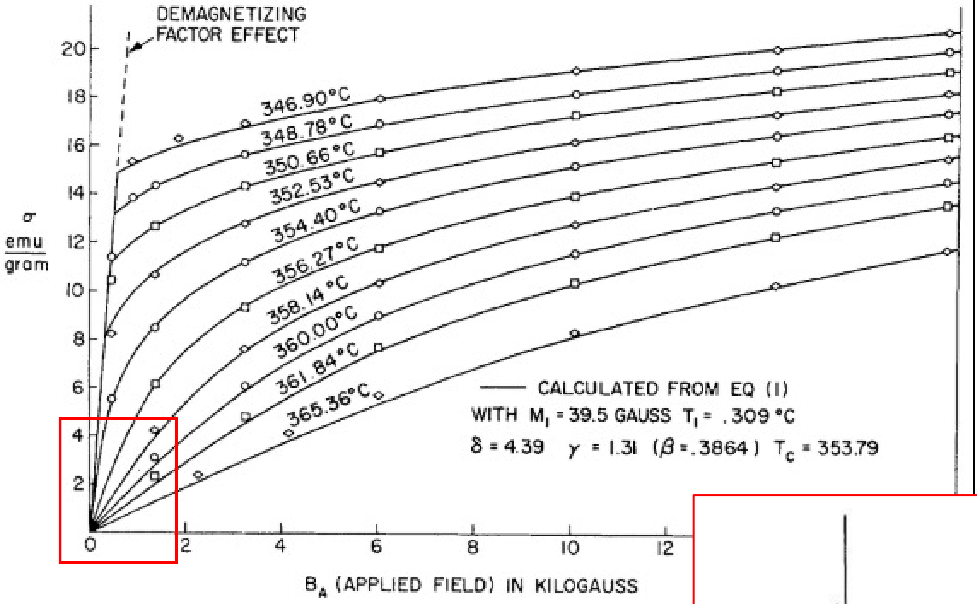


FIG. 2. Raw data of Weiss and Forre
approximate equation of state, Eq. (18).

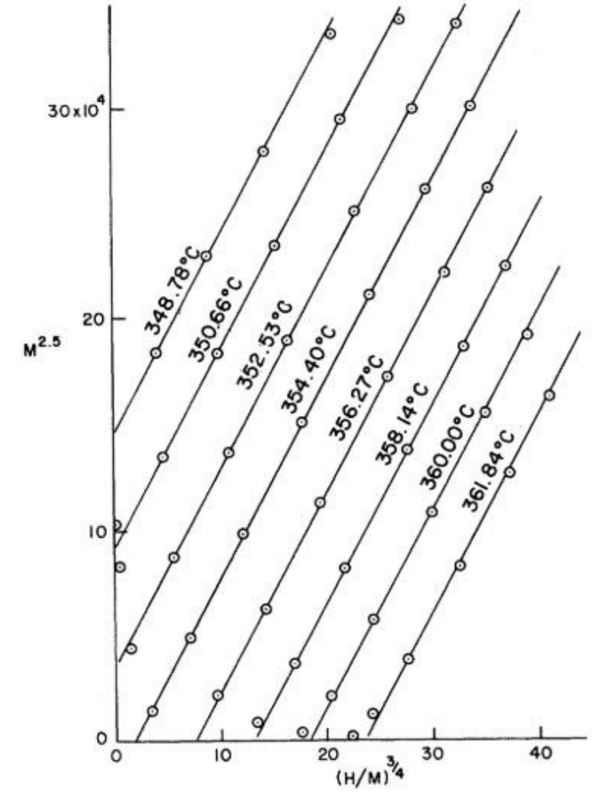


FIG. 3. A replot of the data of Weiss and Forrer to conform to the variables of Eq. (1).

A. Arrott and J. E. Noakes, Phys. Rev. Lett. **19**, 786 (1967).