X-ray intensity from Thomson scattering

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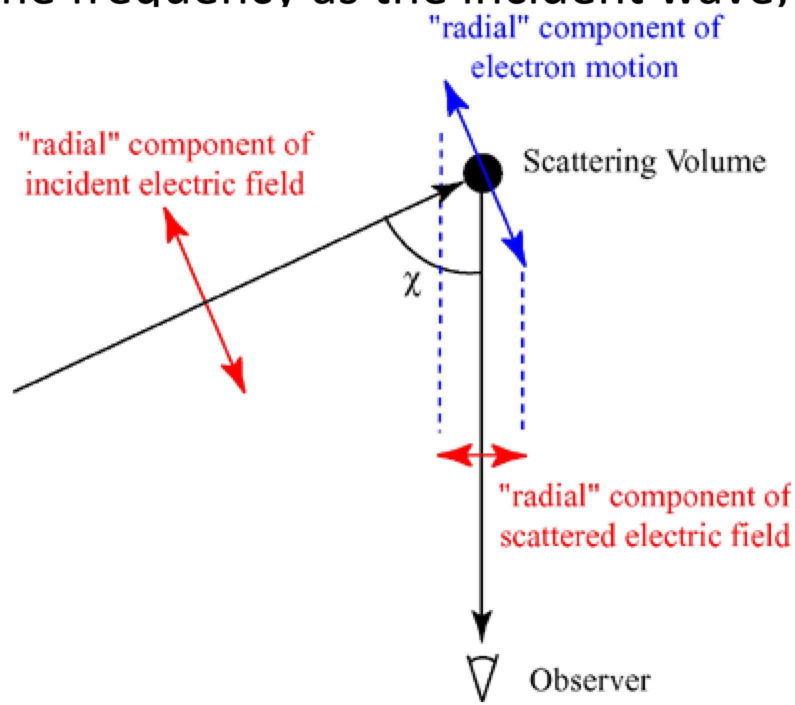
08-24-2017

Thomson scattering

- Elastic.
- Photon scattered by free charged particles.
- In the low-energy limit, the electric field of the incident wave (photon) accelerates the charged particle, causing it, in turn, to emit radiation at the same frequency as the incident wave, and thus the wave is scattered.



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How to calculate scattering ratio (transition probability)?

- Vector potential
  - $A(x,t) = \sum_k \sum_r \left(\frac{\hbar c^2}{2Vw_k}\right)^{\frac{1}{2}} \varepsilon_r(k)[a_r(k,t)e^{ikx} + a_r^*(k,t)e^{-ikx}]$
  - For incident photon with momentum ђk, and polarization $\varepsilon_\alpha(k)$,

  the matrix for transition is

$$\left\langle k',\beta \middle| \frac{e^2}{2mc} A^2(0,t) \middle| k,\alpha \right\rangle = \frac{e^2\hbar}{2mV(\omega\omega')^{1/2}} \varepsilon_\alpha(k)\cdot\varepsilon_\beta(k')e^{i(\omega'-\omega)t}$$

So

Transition rate in solid angle dΩ:

$$\omega_{\alpha\to\beta}(k')d\Omega = \frac{2\pi}{\hbar}\int \frac{Vk'^2dk'd\Omega}{(2\pi)^3}\delta(\hbar\omega'-\omega)\left(\frac{e^2\hbar}{2mV(\omega\omega')^{\frac{1}{2}}}\right)^2 [\varepsilon_\alpha(k)\cdot\varepsilon_\beta(k')]^2$$

$$= \frac{c}{V} r_0^2 [\varepsilon_\alpha(k)\cdot\varepsilon_\beta(k')]^2 \, d\Omega = \frac{c}{V} r_0^2 cos^2\chi \, d\Omega$$

Things to affect Xray reflection intensity

- 1. Multiplicity, the higher multiplicity, the stronger intensity

Table 3.3. Plane Multiplicity Factors, M_{hkl}

System	*hkl*	*hhl*	*hh*0	0*kk*	*hhh*	*hk*0	*h*0*l*	0*kl*	*h*00	0*k*0	00*l*
Cubic	48[a]	24	12	(12)	8	24[a]	(24[a])	(24[a])	6	(6)	(6)
Tetragonal	16[a]	8	4	(8)	(8)	8[a]	8	(8)	4	(4)	2
Hexagonal	24[a]	12[a]	6	(12)	(12)	12[a]	(12[a])	12[a]	6	(6)	2
Orthorhombic	8	(8)	(8)	(8)	(8)	4	(4)	(4)	2	(2)	(2)
Monoclinic	4	(4)	(4)	(4)	(4)	(4)	(2)	(4)	2	(2)	(2)
Triclinic	2	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)

[a] When all permutations of indices do not produce equivalent planes, *M* must be reduced by half.

- 2. Lorentz correction: $\mathbf{I/(sin^2\theta cos\theta)}$ due to intersection between reciprocal lattice and diffractometer circle.
- 3. Absorption.
- 4. monochromater polarization.

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