


A red L-shaped graphic element consisting of a vertical line on the left and a horizontal line at the top, both meeting at a corner on the left side.

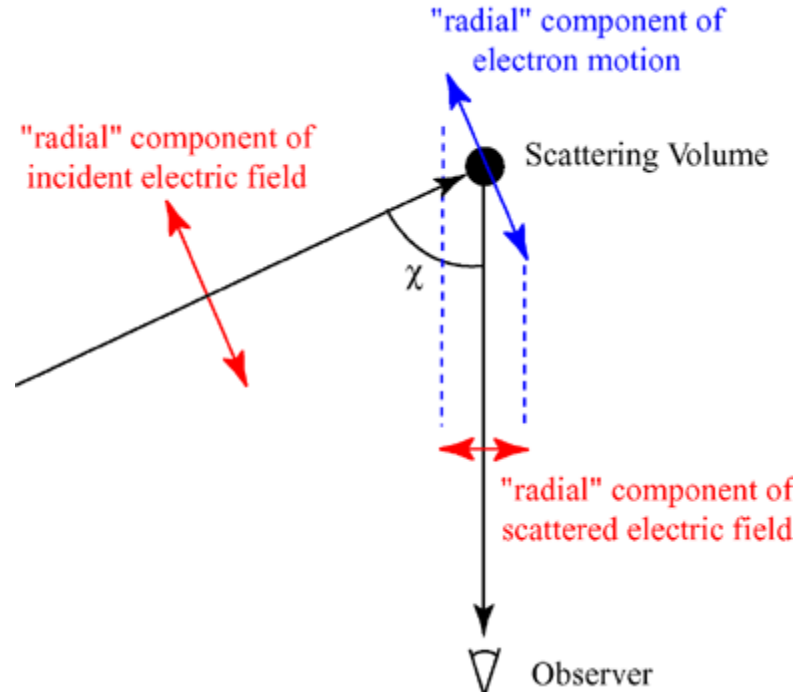
# X-ray intensity from Thomson scattering

A photograph of a paved path lined with trees showing vibrant autumn foliage in shades of yellow, orange, and red. Three people are walking away from the camera down the path. The image is partially obscured by a semi-transparent grey bar.

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# Thomson scattering

- Elastic.
- Photon scattered by free charged particles.
- In the low-energy limit, the [electric](#) field of the incident wave (photon) accelerates the charged particle, causing it, in turn, to emit [radiation](#) at the same frequency as the incident wave, and thus the wave is scattered.



# How to calculate scattering ratio (transition probability)?

- Vector potential

- $A(x, t) = \sum_k \sum_r \left( \frac{\hbar c^2}{2V\omega_k} \right)^{\frac{1}{2}} \varepsilon_r(k) [a_r(k, t)e^{ikx} + a_r^*(k, t)e^{-ikx}]$

- For incident photon with momentum  $\hbar k$ , and polarization  $\varepsilon_\alpha(k)$ , the matrix for transition is

$$\left\langle k', \beta \left| \frac{e^2}{2mc} A^2(0, t) \right| k, \alpha \right\rangle = \frac{e^2 \hbar}{2mV(\omega\omega')^{1/2}} \varepsilon_\alpha(k) \cdot \varepsilon_\beta(k') e^{i(\omega' - \omega)t}$$

So

Transition rate in solid angle  $d\Omega$ :

$$\begin{aligned} \omega_{\alpha \rightarrow \beta}(k') d\Omega &= \frac{2\pi}{\hbar} \int \frac{V k'^2 dk' d\Omega}{(2\pi)^3} \delta(\hbar\omega' - \omega) \left( \frac{e^2 \hbar}{2mV(\omega\omega')^{1/2}} \right)^2 [\varepsilon_\alpha(k) \cdot \varepsilon_\beta(k')]^2 \\ &= \frac{c}{V} r_0^2 [\varepsilon_\alpha(k) \cdot \varepsilon_\beta(k')]^2 d\Omega = \frac{c}{V} r_0^2 \cos^2 \chi d\Omega \end{aligned}$$

# Things to affect Xray reflection intensity

- 1. Multiplicity, the higher multiplicity, the stronger intensity

Table 3.3. Plane Multiplicity Factors,  $M_{hkl}$

System	$hkl$	$hhl$	$hh0$	$0kk$	$hhh$	$hk0$	$h0l$	$0kl$	$h00$	$0k0$	$00l$
Cubic	48 <sup>a</sup>	24	12	(12)	8	24 <sup>a</sup>	(24 <sup>a</sup> )	(24 <sup>a</sup> )	6	(6)	(6)
Tetragonal	16 <sup>a</sup>	8	4	(8)	(8)	8 <sup>a</sup>	8	(8)	4	(4)	2
Hexagonal	24 <sup>a</sup>	12 <sup>a</sup>	6	(12)	(12)	12 <sup>a</sup>	(12 <sup>a</sup> )	12 <sup>a</sup>	6	(6)	2
Orthorhombic	8	(8)	(8)	(8)	(8)	4	(4)	(4)	2	(2)	(2)
Monoclinic	4	(4)	(4)	(4)	(4)	(4)	(2)	(4)	2	(2)	(2)
Triclinic	2	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)	(2)

<sup>a</sup>When all permutations of indices do not produce equivalent planes,  $M$  must be reduced by half.

- 2. Lorentz correction:  $1/(\sin^2\theta\cos\theta)$  due to intersection between reciprocal lattice and diffractometer circle.
- 3. Absorption.
- 4. monochromater polarization.