Stoner Wohlfarth model in 3 dimension

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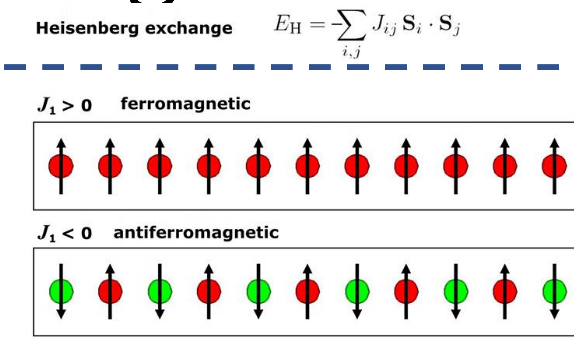
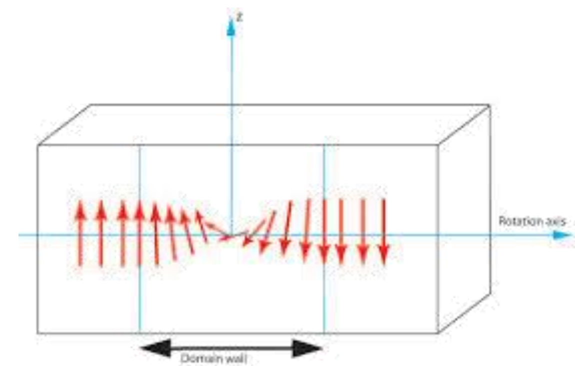
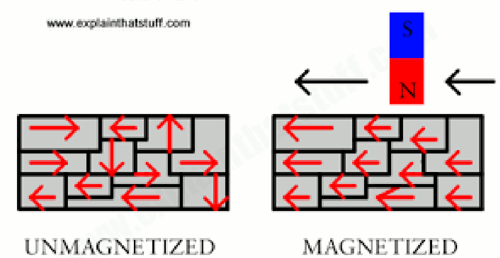
Levels of details for ferromagnets



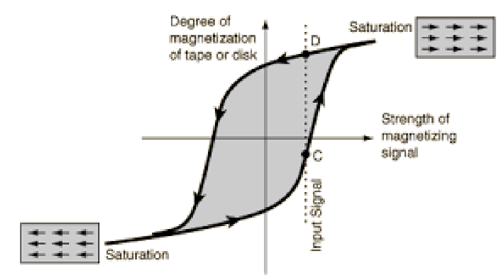
- Atomic level:
  - Exchange interaction that aligns atomic moments $J_{ij}\overrightarrow{S_i} \cdot \overrightarrow{S_j}$
- Micromagnetic level
  - Smear the individual atoms into continuum, see magnetization as a function of position (domain wall)



- Domain level
  - Domains are separated by walls of zero thickness

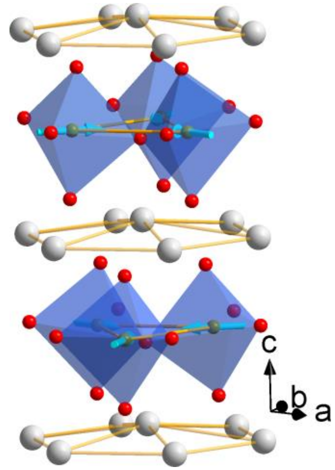


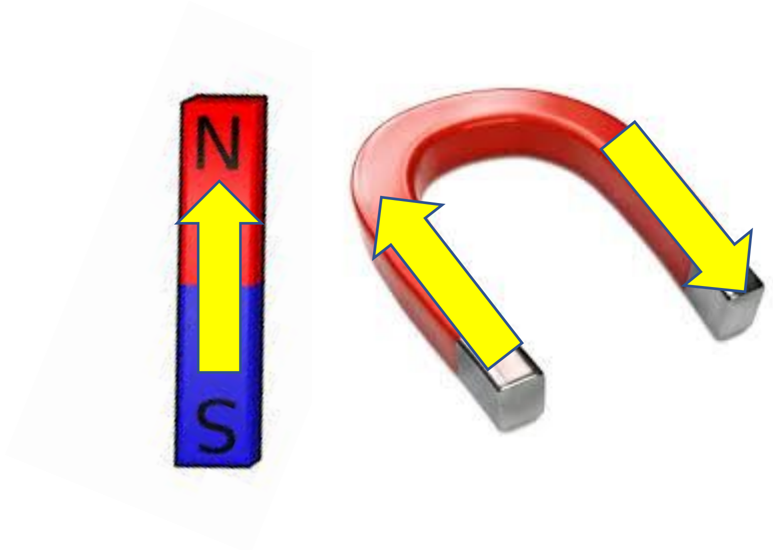
- Nonlinear level
  - Average magnetization of the entire magnet



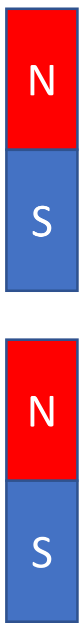
Magnetic anisotropy

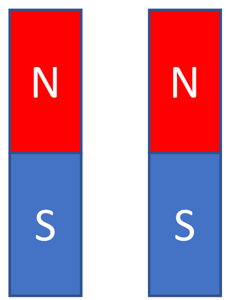
- Magneto-crystalline anisotropy
  - Microscopic
  - Single-ion
  - Symmetry of the atomic local environment
- Shape anisotropy
  - Macrocopic
  - Shape of the magnet
  - Depolarization field





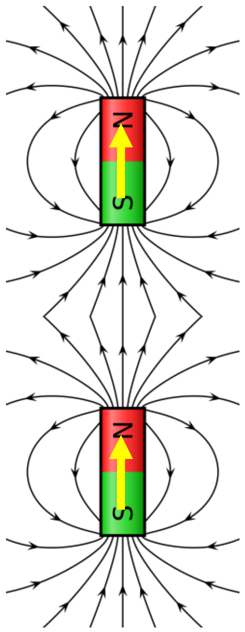
Magnetic shape anisotropy



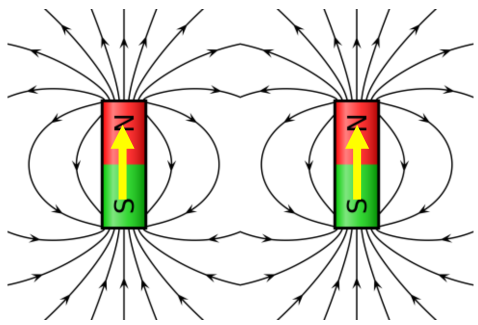


Repulsion,
high energy

Attraction,
low energy



Polarize
each other,
low energy



Depolarize
each other,
high energy

Spheroidal model for anisotropy

Mathematically, both magneto-crystalline anisotropy and magnetic shape anisotropy can be described using the anisotropy tensor (symmetric matrix):

$$\overleftrightarrow{\boldsymbol{D}} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{bmatrix}$$

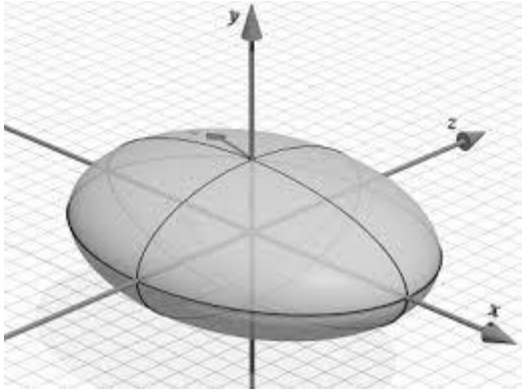
The combined matrix can be diagonalized and along the principle axis, the tensor looks like

$$\overleftrightarrow{\boldsymbol{D}} = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}.$$

Geometrically, these matrices can be described using ellipsoids.

A simplified case assumes the ellipsoid is spherioid (uniaxial).

$$D_{11} = D_{22}$$



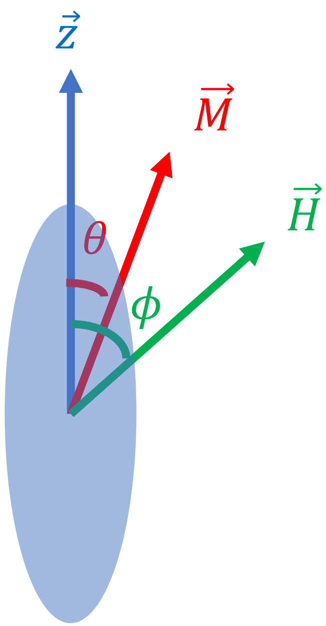
Anisotropy energy:

$$E_A = -\vec{M} \cdot \overleftrightarrow{\boldsymbol{D}} \cdot \vec{M} = -\mathrm{M}^2 \sin^2\theta D_{11} - M^2 \cos^2\theta\, D_{22}$$
$$= -M^2 \sin^2\theta\,(D_{11} - D_{22}) - M^2 D_{22}$$

$$\boxed{E_A = K \sin^2\theta\,, K = -M^2(D_{11} - D_{22})}$$

$$\vec{M} = (\sin\theta\,, \cos\theta\,, 0)$$

Stoner Wohlfarth model: single domain, homogeneous magnetization



Anisotropy energy:

$$E_A = K\sin^2\theta \, , K = -M^2(D_{11} - D_{22})$$

Zeeman energy:

$$E_Z = -HMcos(\phi - \theta)$$

Total energy:

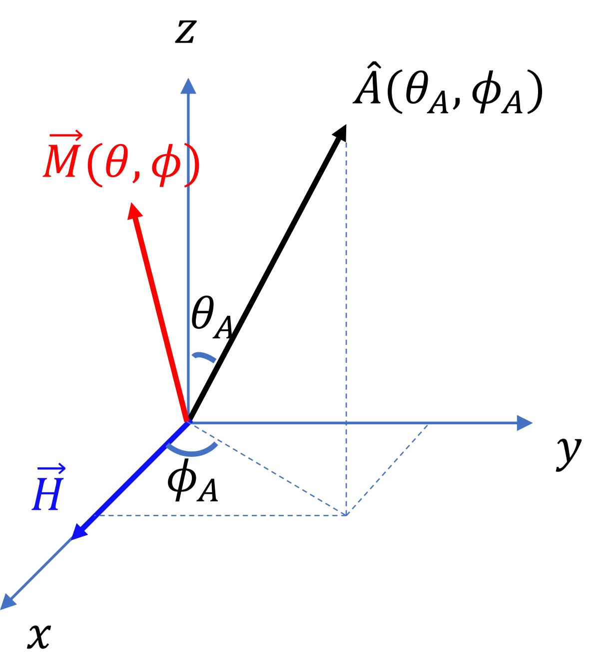
$$E = K\sin^2(\theta) + HMcos(\phi - \theta)$$

- The direction of the **magnetization** is a result of competition between the anisotropy energy and the Zeeman energy.
- Since the magnetic anisotropy is fixed for a sample, we will study the dependence of $\vec{M}$ on H and ϕ.

3 D model

Assume that the angle between $\vec{M}$ and $\hat{A}$ is α.

Assuming uniaxial, the principle axis is represented by vector $\hat{A}(\theta_A, \phi_A)$.



Anisotropy energy:

$$E_A = K\sin^2\alpha$$

$$\cos\alpha = \sin\theta_A\cos\phi_A\sin\theta\cos\phi + \sin\theta_A\sin\phi_A\sin\theta\sin\phi + \cos\theta_A\cos\theta$$

Zeeman energy:

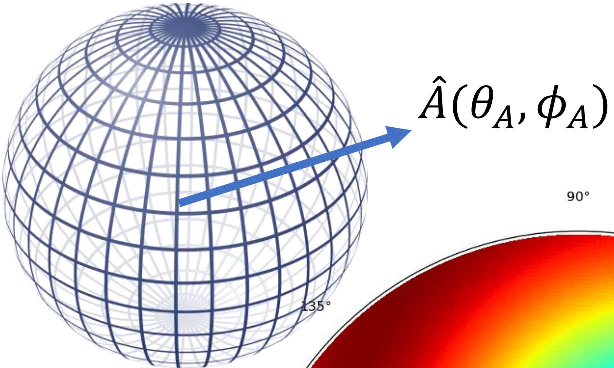
$$E_Z = -\sin\theta\cos\phi\, HM$$

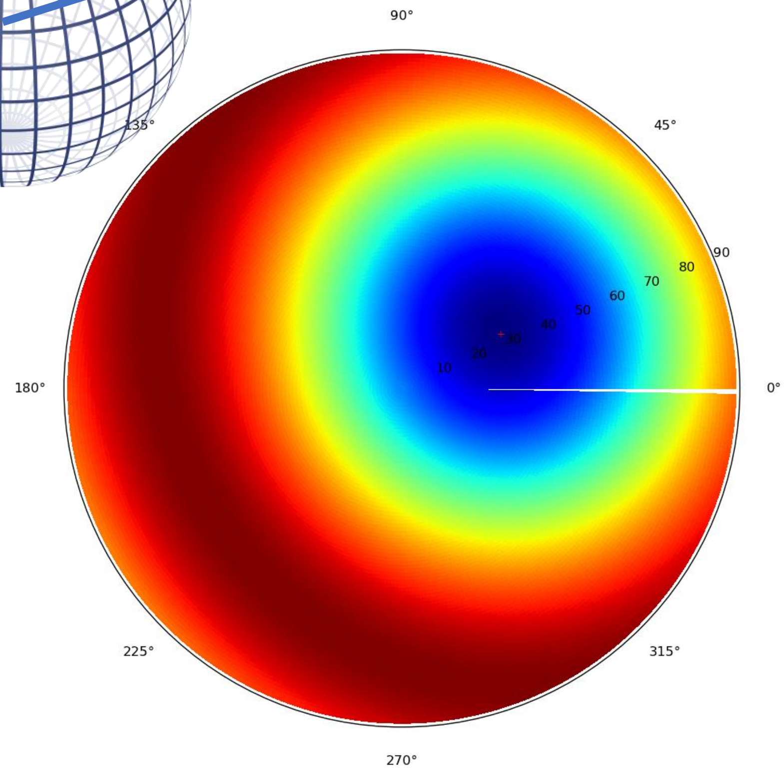
Total energy:

$$E_{total} = K(1-\cos^2\alpha) - \sin\theta\cos\phi\, HM = K\left[(1-\cos^2\alpha) - \boxed{\frac{HM}{K}}\sin\theta\cos\phi\right]$$

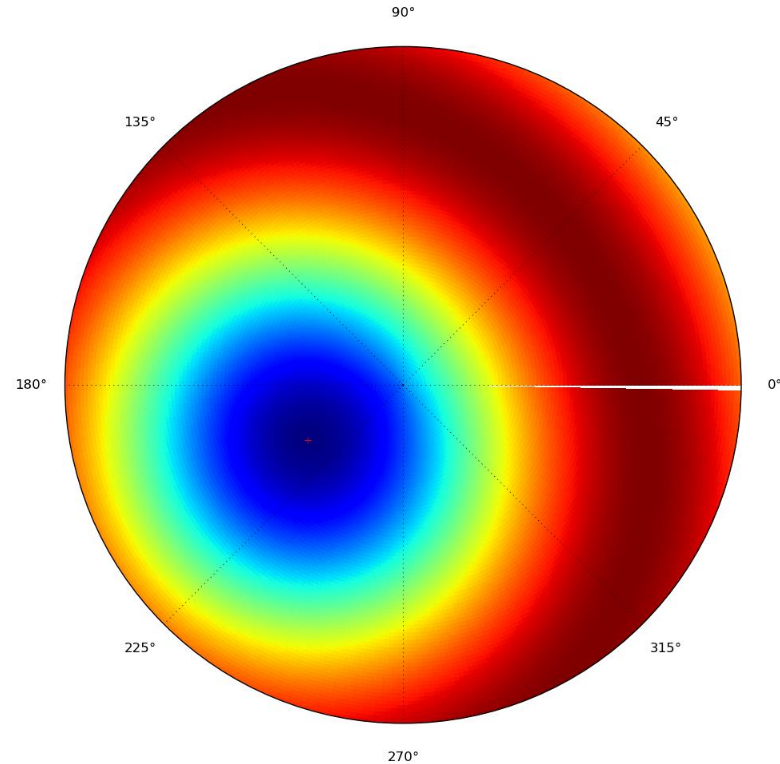
Dimensionless parameter is enough to describe the field strength.

Polar map of energy $\theta = 30, \phi = 30, H = 0.$



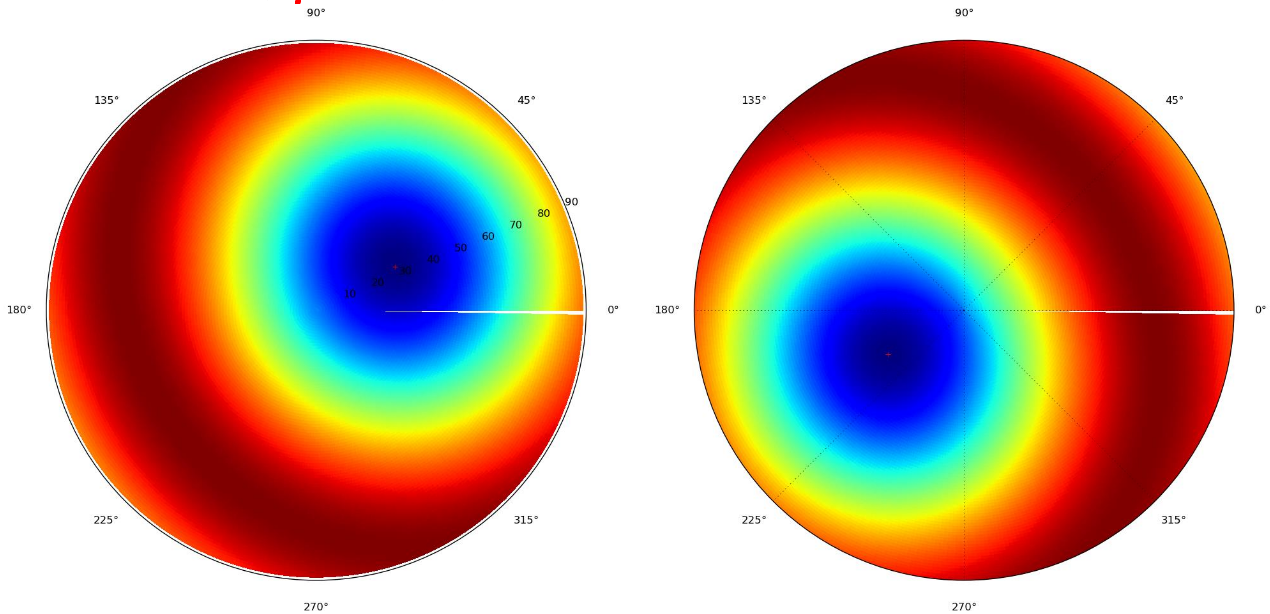


North hemisphere

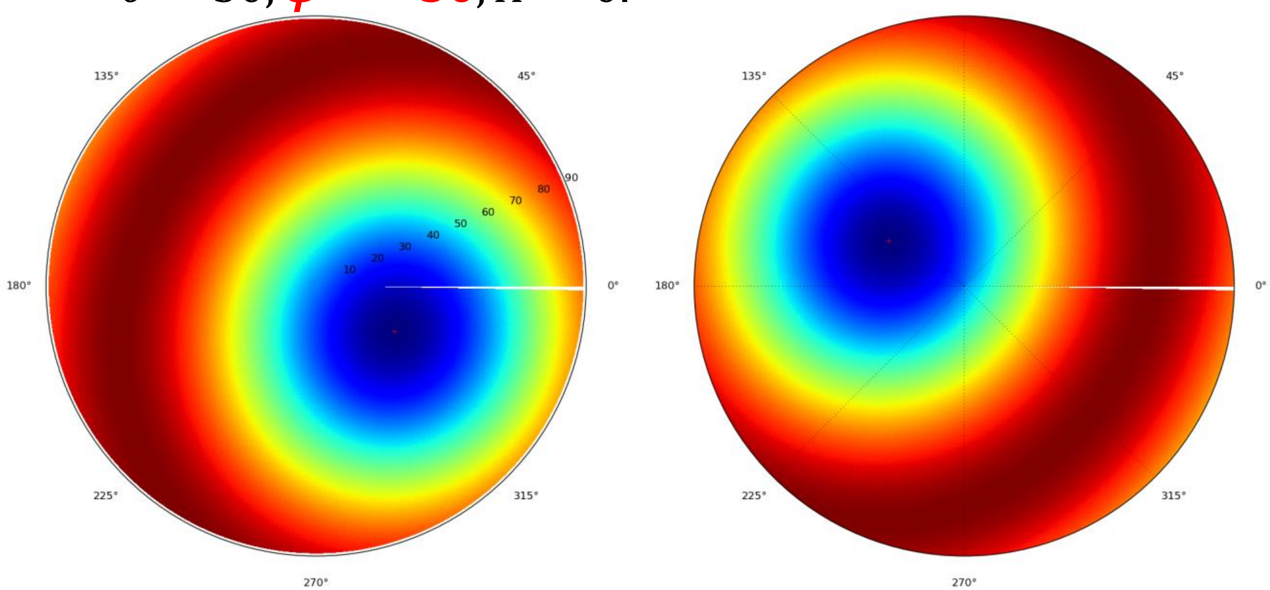


South hemisphere

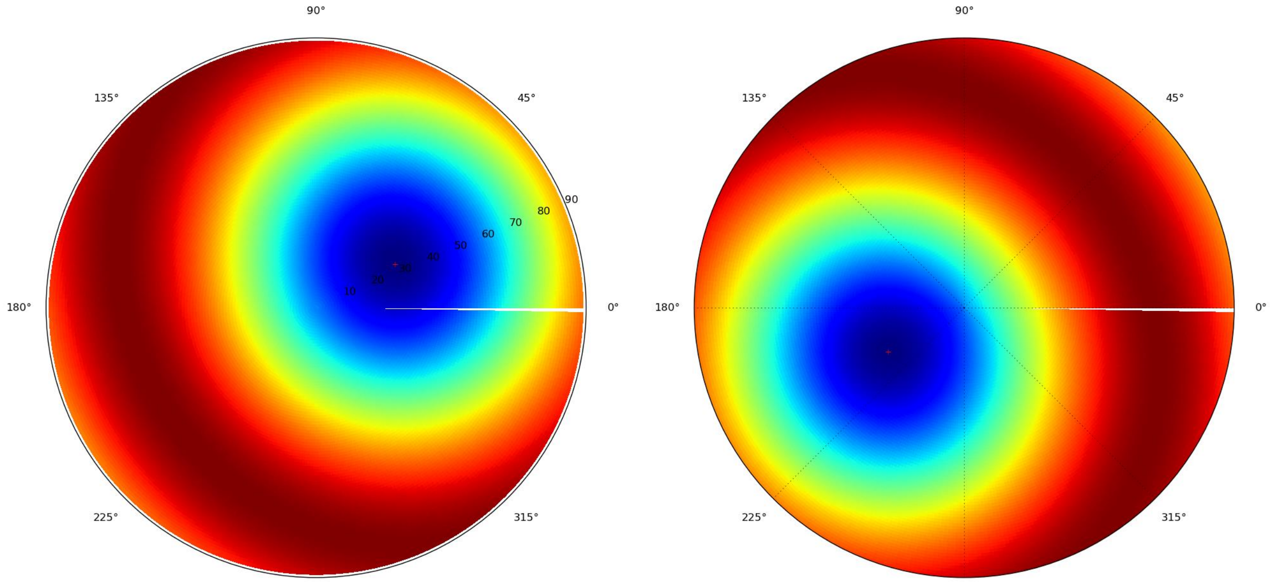
$\theta = 30, \boldsymbol{\phi = 30}, H = 0.$



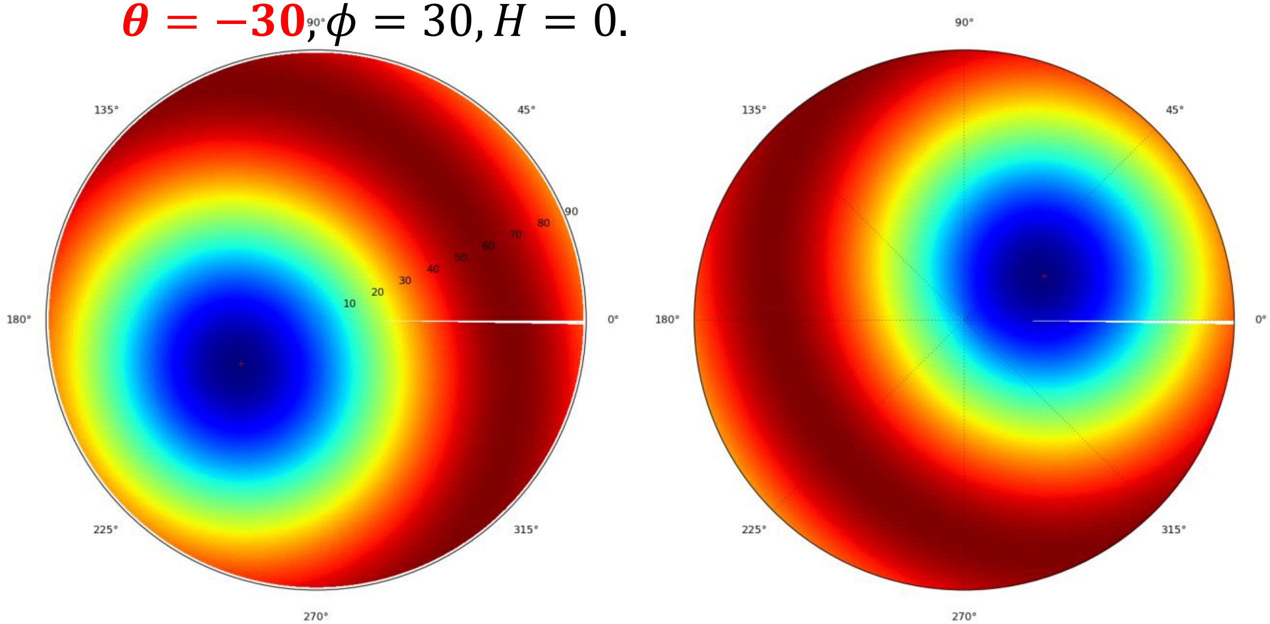
$\theta = 30, \boldsymbol{\phi = -30}, H = 0.$



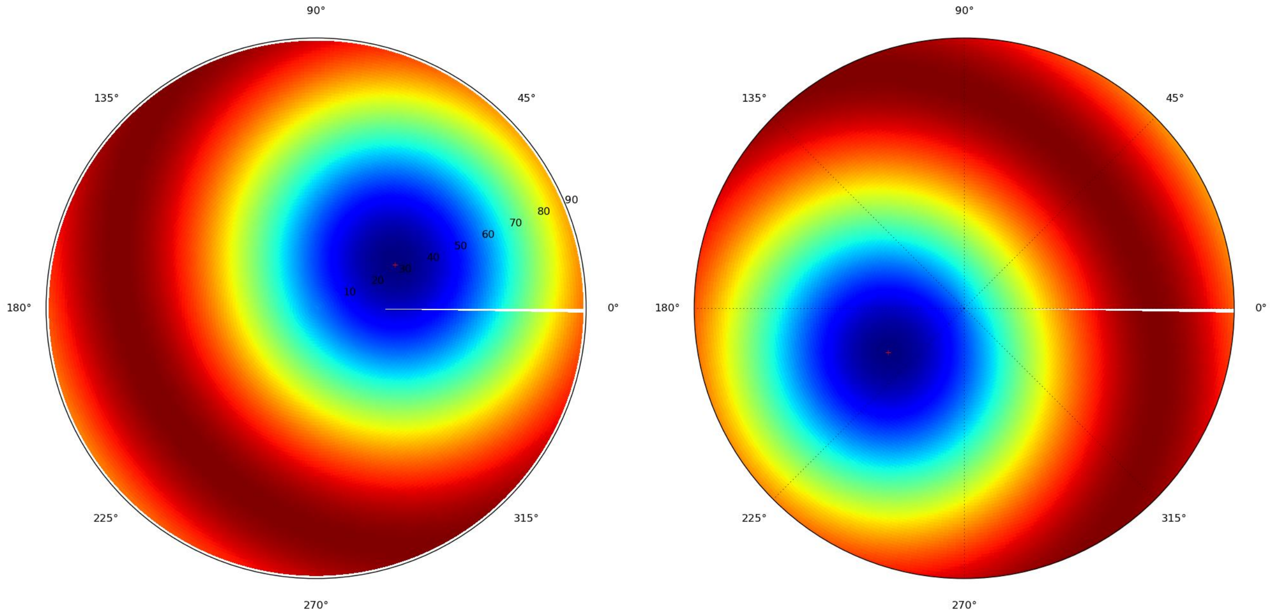
$\boldsymbol{\theta = 30}, \phi = 30, H = 0.$



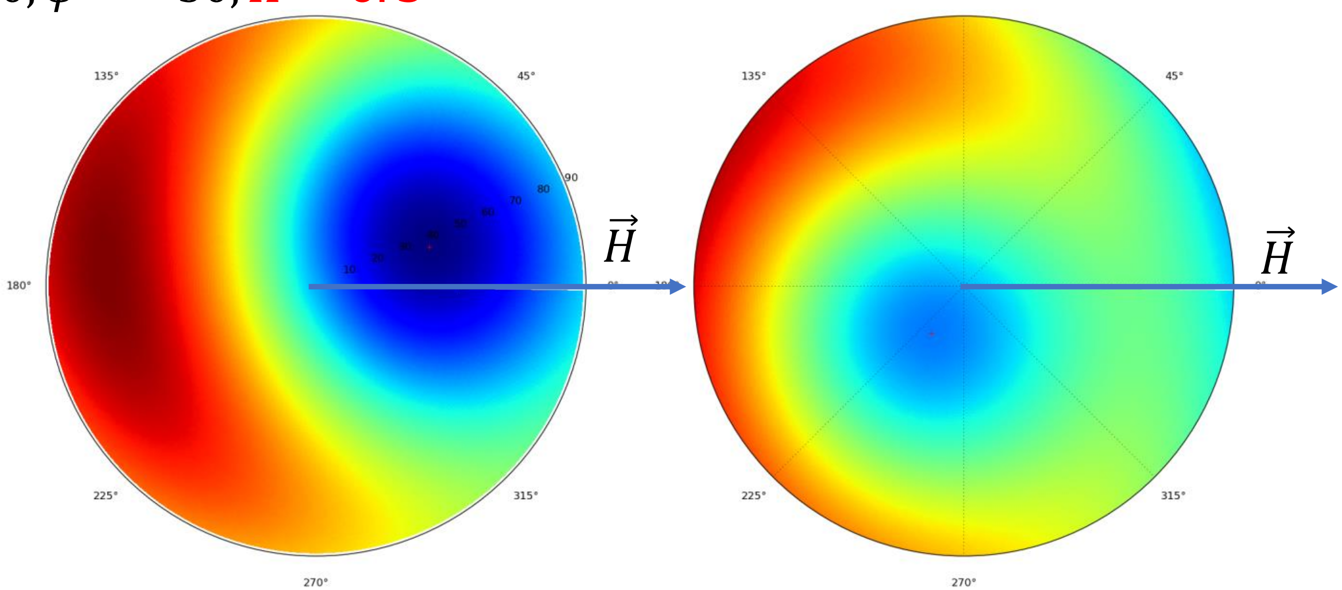
$\boldsymbol{\theta = -30}, \phi = 30, H = 0.$



$\theta = 30, \phi = 30,$ $\boldsymbol{H = 0}$.

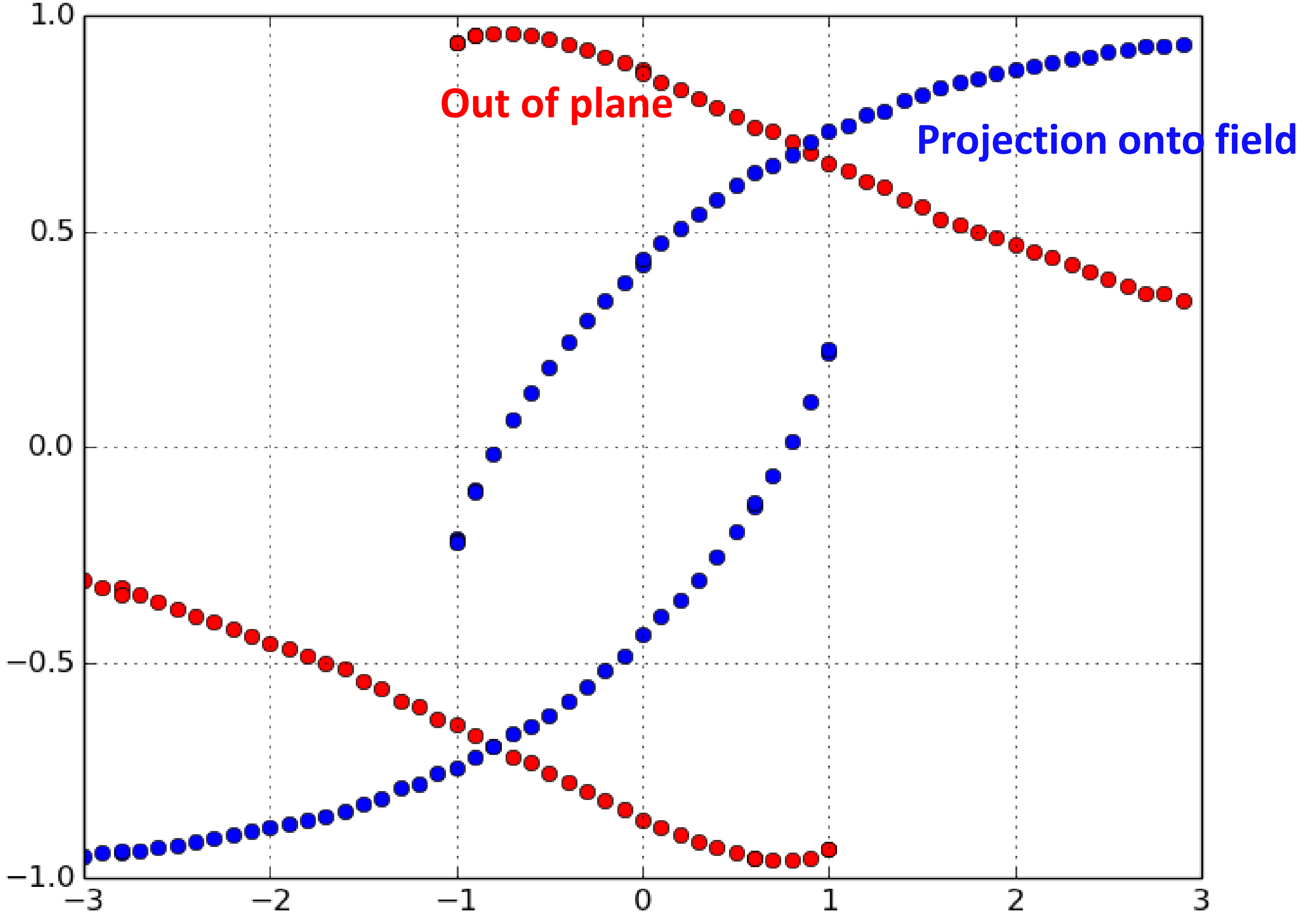


$\theta = 30, \phi = -30,$ $\boldsymbol{H = 0.5}$

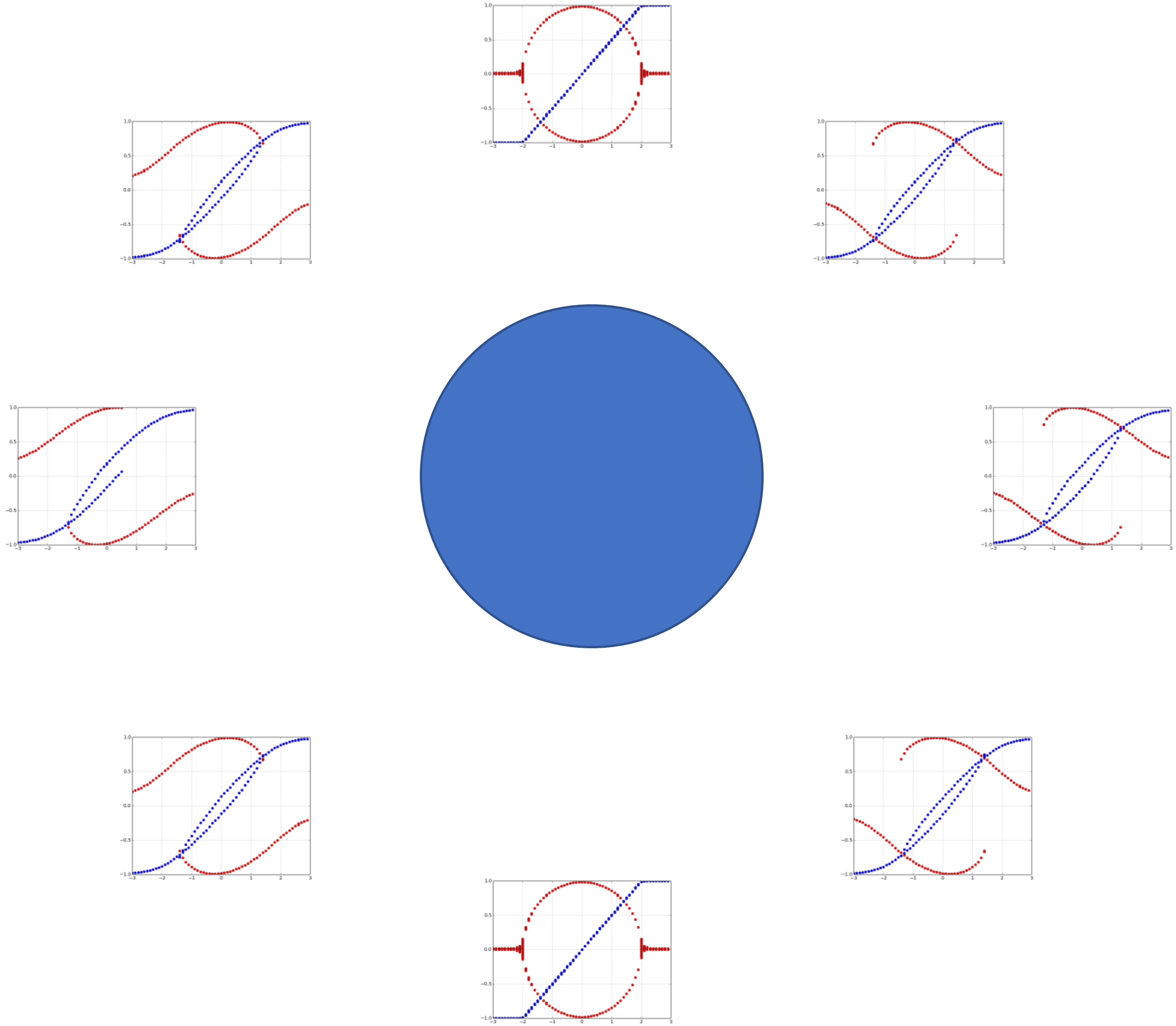


Field dependence (MH loop)

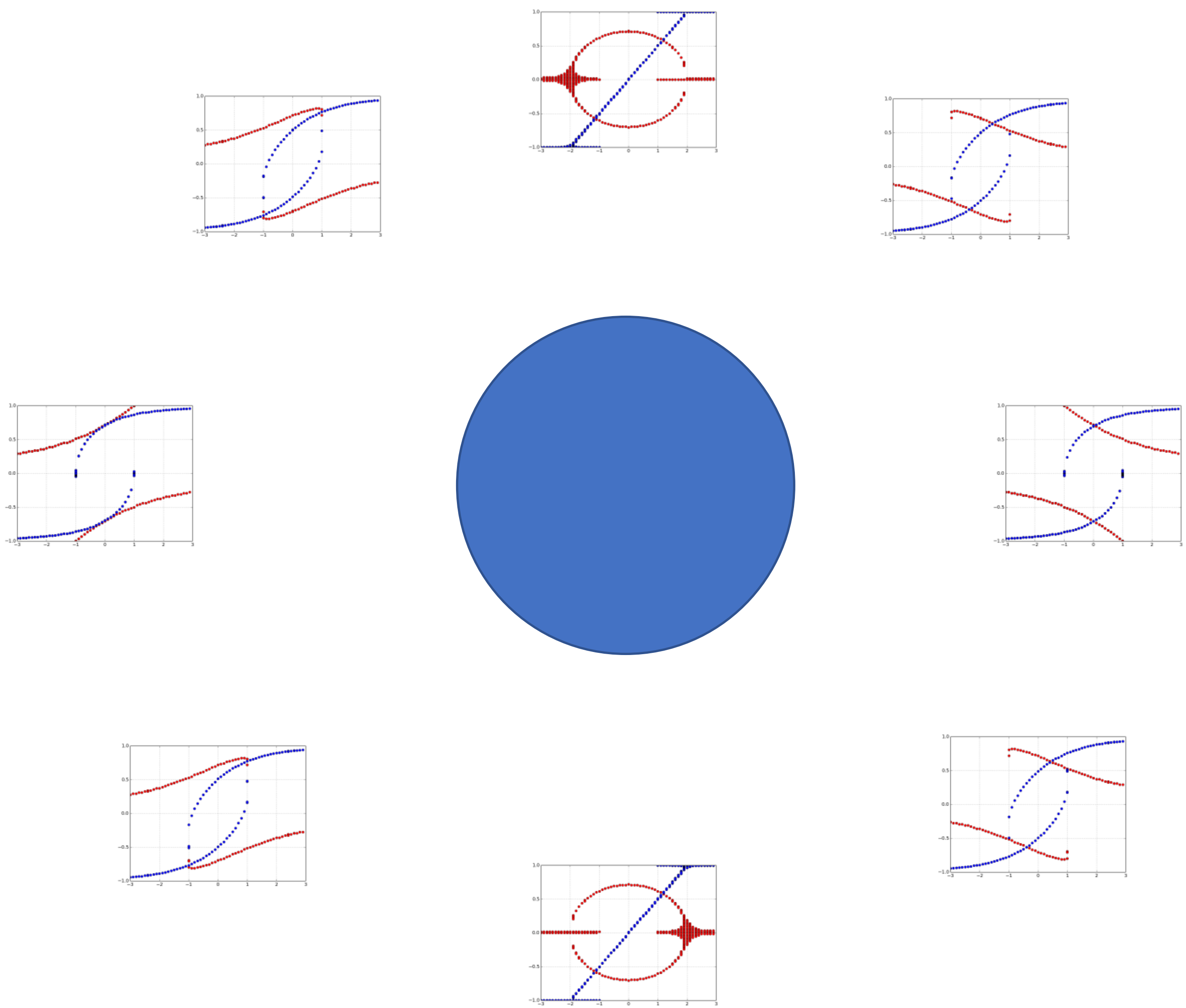
$\theta = 30, \phi = 30$



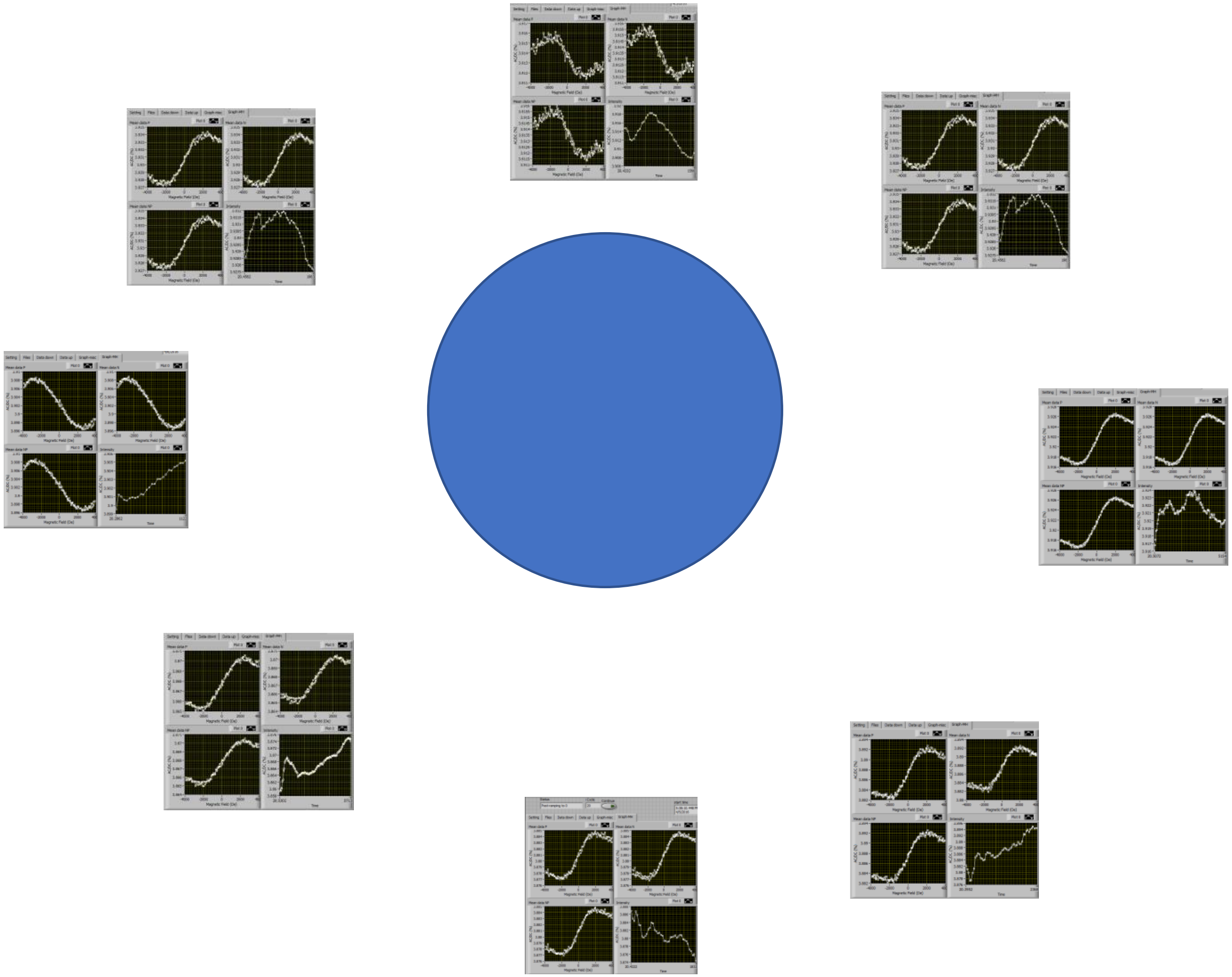
MH loop at different ϕ, $\theta = 10$ °



MH loop at different $\phi, \theta = 45^\circ$



Compare to experiment



Conclusion

- 3 D Stoner Wohlfarth model can be simulated at different uniaxial principle axis direction
- The in-plane and out-of-plane MH loop show different behavior.
- The out-of-plane is similar to what we have observed.