Dirac Equation: Derivation & Solutions

Corbyn Mellinger

Xu Group Meeting

Mar. 10 2017

Weyl Semimetals

New quantum transition in Weyl semimetals with correlated disorder

T. Louvet, D. Carpentier, and A. A. Fedorenko

Phys. Rev. B **95**, 014204 (2017) – Published 6 January 2017

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Editors' Suggestion Rapid Communication

Photocurrents in Weyl semimetals

Ching-Kit Chan, Netanel H. Lindner, Gil Refael, and Patrick A. Lee

Phys. Rev. B **95**, 041104(R) (2017) – Published 13 January 2017

Tilted disordered Weyl semimetals

Maximilian Trescher, Björn Sbierski, Piet W. Brouwer, and Emil J. Berg

Phys. Rev. B **95**, 045139 (2017) – Published 25 January 2017

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Type-II Dirac surface states in topological crystalline insulators

. Chan, Xiao Li, Y. Nohara, and A. P. Schnyder

5151 (2017) – Published 30 January 2017

Anomalous Hall effects beyond Berry magnetic fields in a Weyl metal phase

Iksu Jang, Jae-Ho Han, and Ki-Seok Kim

Phys. Rev. B **95**, 054117 (2017) – Published 21 February 2017

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Editors' Suggestion

Disorder-induced phase transitions of type-II Weyl semimetals

Moon Jip Park, Bora Basa, and Matthew J. Gilbert

Phys. Rev. B **95**, 094201 (2017) – Published 2 March 2017

Weyl Semimetals

TOPOLOGICAL MATTER

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Discovery of a Weyl fermion semimetal and topological Fermi arcs

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A Weyl semimetal is a new state of matter that hosts Weyl fermions as emergent quasiparticles and admits a topological classification that protects Fermi arc surface states on the boundary of a bulk sample. This unusual electronic structure has deep analogies with particle physics and leads to unique topological properties. We report the experimental discovery of a Weyl semimetal, tantalum arsenide (TaAs). Using photoemission spectroscopy, we directly observe Fermi arcs on the surface, as well as the Weyl fermion cones and Weyl nodes in the bulk of TaAs single crystals. We find that Fermi arcs terminate on the Weyl fermion nodes, consistent with their topological character. Our work opens the field for the experimental study of Weyl fermions in physics and materials science.

- Named "Breakthrough of the Year" by Physics Today (2015)
- We should first understand Weyl fermions, to understand these materials

Klein-Gordon Equation

- Attempt to reconcile special relativity with quantum mechanics

- Substitution of *E*, *p* with quantum operators leads to an expression:

$$E^2 = m^2c^4 + p^2c^2$$

$$E \rightarrow i\hbar\frac{\partial}{\partial t}$$

$$p \rightarrow -i\hbar\nabla$$

$$\left(\Box + \mu^2\right)\phi = 0$$

Klein-Gordon Equation

$$\left(\Box + \mu^2\right)\phi = 0$$

$$\Box \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

$$\mu \equiv \frac{mc}{\hbar}$$

- Describes spin-0 particles (e.g. pion)
- Unable to describe electrons (spin-1/2)
- Dirac: want a first-order differential equation.
- What happens if we take a square root?

Dirac Equation: Motivation

$$\sqrt{-\Box}\psi = \mu\psi$$

with

$$\sqrt{-\Box} = A\frac{\partial}{\partial x} + B\frac{\partial}{\partial y} + C\frac{\partial}{\partial z} + \frac{i}{c}D\frac{\partial}{\partial t}$$

- These values A, B, C, D can't be scalars
  - Need $A^2=B^2=C^2=D^2=1$, but cross-terms all zero

- Dirac's insight: these can be matrices!

Dirac Matrices

$$A = i\beta\alpha_1,\ B = i\beta\alpha_2,\ C = i\beta\alpha_3,\ D = \beta$$

must satisfy:

$$\alpha_i^2 = \beta^2 = 1,\ \alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij},\ \alpha_i\beta + \beta\alpha_i = 0$$

in order to get each term squared unity, cross-terms zero.

But there are multiple ways to satisfy this…what do they mean?

Dirac Representation

- Makes use of the Pauli spin matrices:

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix}, \ \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$$

Dirac Representation: Implication

- Solutions require two functions: γ and $\bar{\gamma}$
  - Two functions have opposite charges, but other properties are alike

- Implies existence of “anti-particles” with same mass, opposite charge
  - Observed positron (anti-electron) in 1932

Majorana Representation

- Another set of γ-matrices which has necessary conditions:

$$\gamma_M^0 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix},\ \gamma_M^1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix},$$

$$\gamma_M^2 = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix},\ \gamma_M^3 = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & i\sigma_3 \end{pmatrix}$$

- Solutions are equivalent to their own conjugated solution
  - Particle is its *own* antiparticle

Weyl Representation

- Also called "chiral representation" (helicity of solutions is preserved under Lorentz transformation)

$$\gamma_W^0 = \begin{pmatrix} 0 & \sigma_0 \\ \sigma_0 & 0 \end{pmatrix}, \; \gamma_W^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$$

- Conserved helicity requires massless particle: Weyl fermions are massless