Stoner Wohlfarth Model for Magneto Anisotropy

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Levels of details for ferromagnets

• Atomic level:
  • Exchange interaction that aligns atomic moments $J_{ij} \vec{S}_i \cdot \vec{S}_j$

• Micromagnetic level
  • Smear the individual atoms into continuum, see magnetization as a function of position (domain wall)

• Domain level
  • Domains are separated by walls of zero thickness

• Nonlinear level
  • Average magnetization of the entire magnet
Magnetic anisotropy

• Magneto-crystalline anisotropy
  • Microscopic
  • Single-ion
  • Symmetry of the atomic local environment

• Shape anisotropy
  • Macroscopic
  • Shape of the magnet
  • Depolarization field
Magnetic shape anisotropy

Attraction, low energy

Repulsion, high energy

Depolarization factor:

$$D_z = \frac{1}{\alpha^2 - 1} \left[ \frac{\alpha}{\sqrt{\alpha^2 - 1}} \ln(\alpha + \sqrt{\alpha^2 - 1}) - 1 \right]$$

$$D_x + D_y + D_z = 1$$

$\alpha > 1$ is the aspect ratio $\frac{L_z}{L_x}$
Spheroidal model for anisotropy

**Mathematically**, both magneto-crystalline anisotropy and magnetic shape anisotropy can be described using the anisotropy tensor (symmetric matrix):

\[
\mathbf{D} = \begin{bmatrix}
D_{xx} & D_{xy} & D_{xz} \\
D_{xy} & D_{yy} & D_{yz} \\
D_{xz} & D_{yz} & D_{zz}
\end{bmatrix}
\]

The combined matrix can be diagonalized and along the principle axis, the tensor looks like

\[
\mathbf{D} = \begin{bmatrix}
D_{11} & 0 & 0 \\
0 & D_{22} & 0 \\
0 & 0 & D_{33}
\end{bmatrix}
\]

**Geometrically**, these matrices can be described using ellipsoids.

A simplified case assumes the ellipsoid is spherioid. 

\[D_{11} = D_{22}\]

Anisotropy energy:

\[
E_A = -\mathbf{M} \cdot \mathbf{D} \cdot \mathbf{M} = -M^2 \sin^2 \theta D_{11} - M^2 \cos^2 \theta D_{33} \\
= -M^2 \sin^2 \theta (D_{11} - D_{33}) - M^2 D_{22}
\]

\[E_A = K \sin^2 \theta, K = -M^2 (D_{11} - D_{33})\]
Stoner Wohlfarth model: single domain, homogeneous magnetization

Anisotropy energy:

\[ E_A = K \sin^2 \theta, \quad K = -M^2(D_{11} - D_{22}) \]

Zeeman energy:

\[ E_Z = -HM\cos(\phi - \theta) \]

Total energy:

\[ E = K \sin^2(\theta) + HM\cos(\phi - \theta) \]

- The direction of the magnetization is a result of competition between the anisotropy energy and the Zeeman energy.
- Since the magnetic anisotropy is fixed for a sample, we will study the dependence of \( \vec{M} \) on \( H \) and \( \phi \).
Stoner Wohlfarth model: critical field

Total energy:

\[ E = K \sin^2(\theta) + HM \cos(\phi - \theta) \]

- Assuming a nonzero \( \phi \) (here 15°), we can plot the angular dependence of total energy.

- As field increases, the energy of the two minima become different.
- One minimum eventually disappears at high field, corresponding to saturation magnetization.
Stoner Wohlfarth model: random field angle example

Total energy: \( E = K \sin^2(\theta) + HM \cos(\theta - \phi) \)

- At high enough field one minimum disappears.
  Mathematically, this corresponds to:
  \[
  \frac{\partial E}{\partial \theta} = 0, \quad \frac{\partial^2 E}{\partial^2 \theta} = 0
  \]

\[
K \sin 2\theta + HM \sin(\phi - \theta) = 0
\]
\[
2K \cos 2\theta - HM \cos(\phi - \theta) = 0
\]

Expand:
\[
\frac{2K}{M} \sin \theta \cos \theta + H_x \cos \theta - H_z \sin \theta = 0
\]
\[
\frac{2K}{M} \cos^2 \theta - \sin^2 \theta - (H_z \cos \theta + H_x \sin \theta) = 0
\]

\[
H_x = H \sin \theta; \quad H_z = H \cos \theta
\]
Stoner Wohlfarth model: random field angle example

Total energy: \( E = K \sin^2(\theta) + HM \cos(\theta - \phi) \)

Expand:
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\frac{2K}{M} \sin \theta \cos \theta + H_x \cos \theta - H_z \sin \theta = 0
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\[
\frac{2K}{M} \sin \theta \cos \theta + H_x \cos \theta - H_z \sin \theta = 0 \quad [\times \sin \theta]
\]
\[
\frac{2K}{M} \cos^2 \theta - \sin^2 \theta - (H_z \cos \theta + H_x \sin \theta) = 0 \quad [\times \cos \theta]
\]

\[
H_z = \frac{2K}{M} \cos^3 \theta
\]
\[
H_x = -\frac{2K}{M} \sin^3 \theta
\]

\[
\frac{2}{H_x^3} + \frac{2}{H_z^3} = \left(\frac{2K}{M}\right)^{\frac{2}{3}}
\]

\[
H_c = \frac{2K}{M}
\]
Slonczewski asteroid
Hysteresis
Angular dependence (anisotropic MR)
Conclusion

• Stoner Wohlfarth model describes the magnetic anisotropy problem of a single domain particle
• It can explain the ideal case hysteresis and angular dependence of magnetizations.