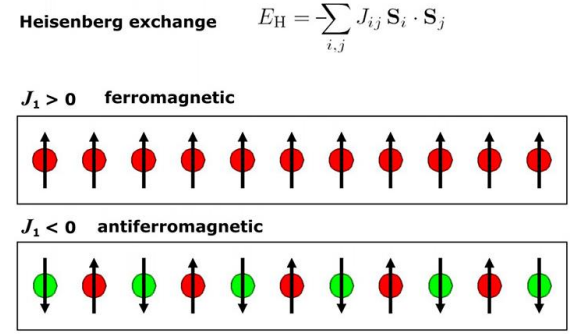
Stoner Wohlfarth Model for Magneto Anisotropy

Xiaoshan Xu

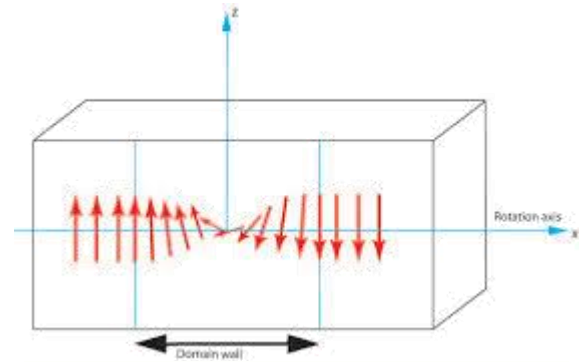
2016/10/20

Levels of details for ferromagnets

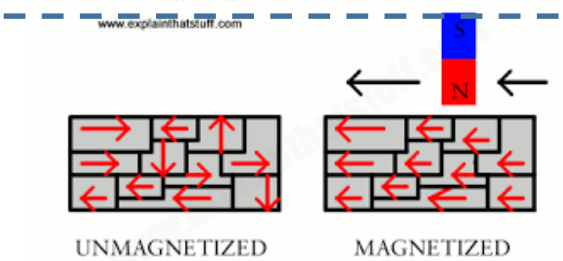
- Atomic level:
  - Exchange interaction that aligns atomic moments $J_{ij}\overrightarrow{S_i} \cdot \overrightarrow{S_j}$



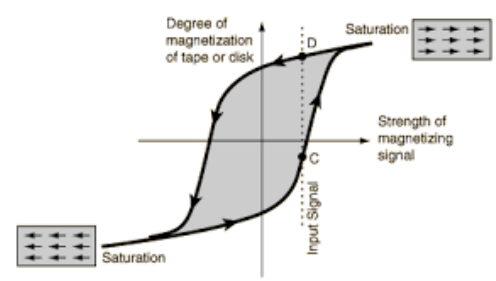
- Micromagnetic level
  - Smear the individual atoms into continuum, see magnetization as a function of position (domain wall)



- Domain level
  - Domains are separated by walls of zero thickness

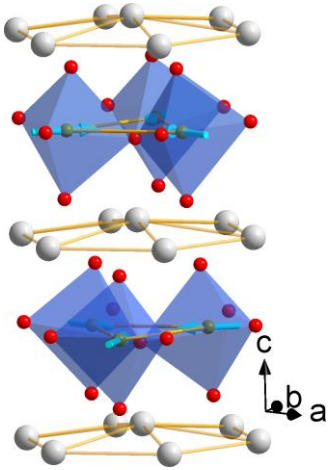


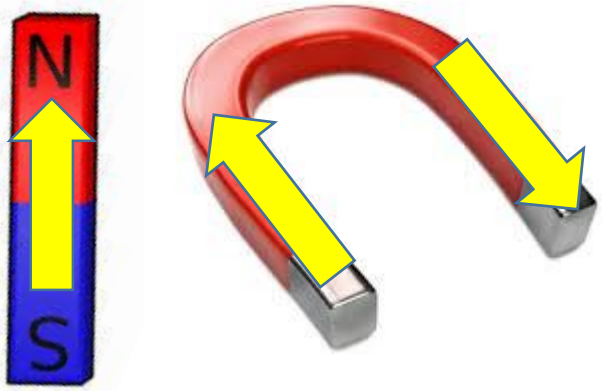
- Nonlinear level
  - Average magnetization of the entire magnet



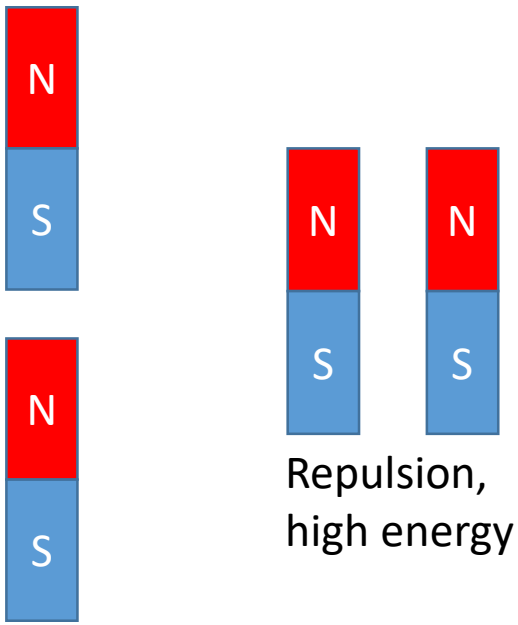
Magnetic anisotropy

- Magneto-crystalline anisotropy
  - Microscopic
  - Single-ion
  - Symmetry of the atomic local environment
- Shape anisotropy
  - Macrocopic
  - Shape of the magnet
  - Depolarization field



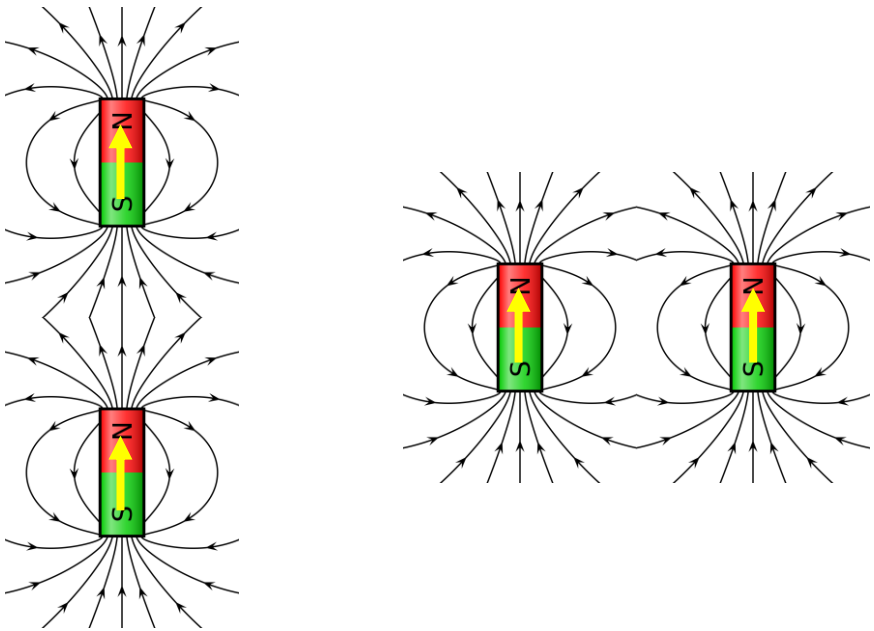


Magnetic shape anisotropy



Attraction,
low energy

Repulsion,
high energy



Polarize
each other,
low energy

Depolarize
each other,
high energy

Depolarization factor:

$$D_z = \frac{1}{\alpha^2-1}\left[\frac{\alpha}{\sqrt{\alpha^2-1}}\ln\left(\alpha+\sqrt{\alpha^2-1}\right)-1\right]$$

$$D_x + D_y + D_z = 1$$

$\alpha > 1$ is the aspect ratio $\frac{L_z}{L_x}$

Spheroidal model for anisotropy

Mathematically, both magneto-crystalline anisotropy and magnetic shape anisotropy can be described using the anisotropy tensor (symmetric matrix):

$$\overleftrightarrow{\boldsymbol{D}} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{bmatrix}$$

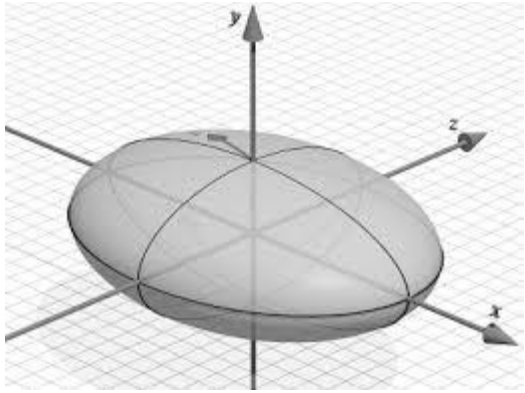
The combined matrix can be diagonalized and along the principle axis, the tensor looks like

$$\overleftrightarrow{\boldsymbol{D}} = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}.$$

Geometrically, these matrices can be described using ellipsoids.

A simplified case assumes the ellipsoid is spherioid.

$$D_{11} = D_{22}$$

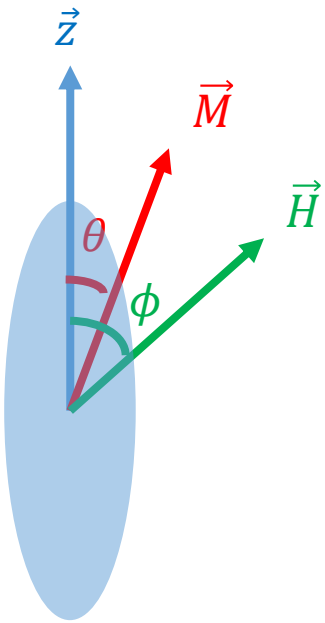


Anisotropy energy:

$$E_A = -\vec{M} \cdot \overleftrightarrow{\boldsymbol{D}} \cdot \vec{M} = -\mathrm{M}^2 \sin^2\theta D_{11} - M^2 \cos^2\theta \, D_{33}$$
$$= -M^2 \sin^2\theta \, (D_{11} - D_{33}) - M^2 D_{22}$$

$$\boxed{E_A = K \sin^2\theta \, , K = -M^2 (D_{11} - D_{33})}$$

Stoner Wohlfarth model: single domain, homogeneous magnetization



Anisotropy energy:

$$E_A = K\sin^2\theta\,, K = -M^2(D_{11} - D_{22})$$

Zeeman energy:

$$E_Z = -HMcos(\phi - \theta)$$

Total energy:

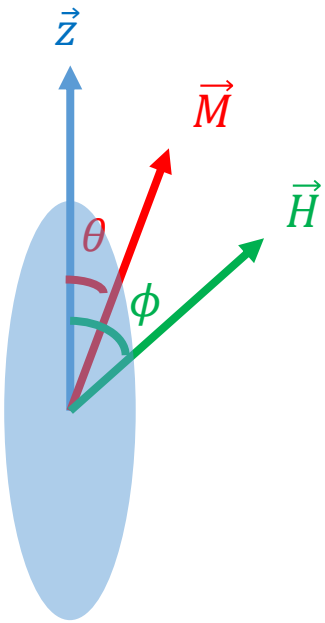
$$E = K\sin^2(\theta) + HMcos(\phi - \theta)$$

- The direction of the **magnetization** is a result of competition between the anisotropy energy and the Zeeman energy.
- Since the magnetic anisotropy is fixed for a sample, we will study the dependence of $\vec{M}$ on H and ϕ.

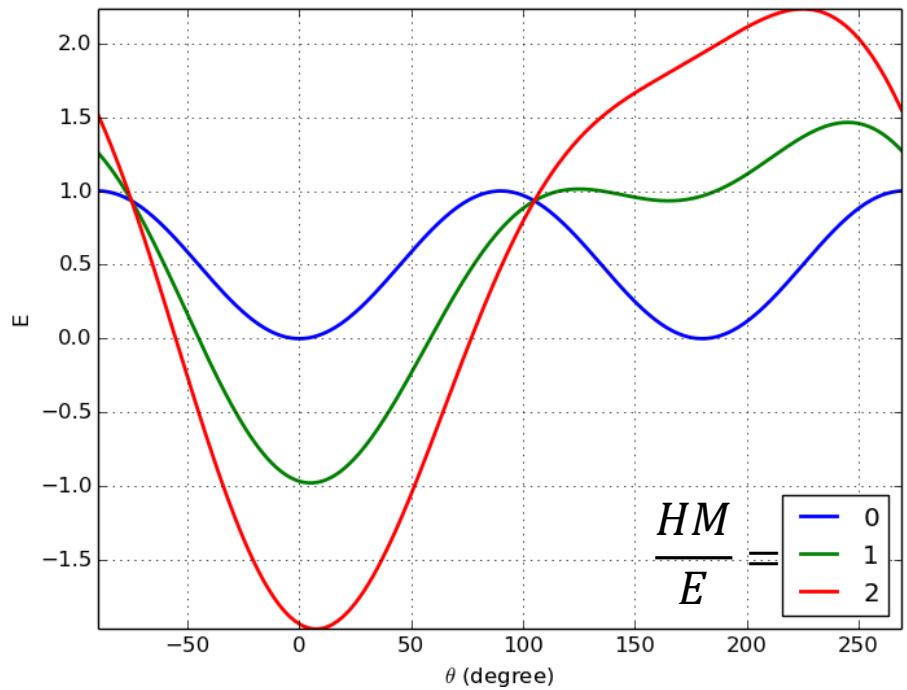
Stoner Wohlfarth model: critical field

Total energy:

$$E = K\sin^2(\theta) + HMcos(\phi - \theta)$$



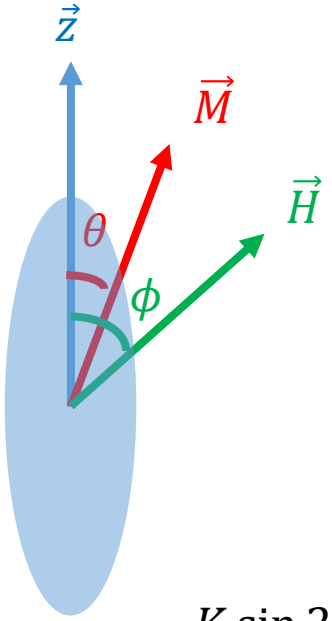
- Assuming a nonzero ϕ (here 15˚), we can plot the angular dependence of total energy.



- As field increases, the energy of the two minima become different.
- One minimum eventually disappears at high field, corresponding to saturation magnetization.

Stoner Wohlfarth model: random field angle example

$$\boxed{\text{Total energy: } E = K\sin^2(\theta) + HM\cos(\theta - \phi)}$$



- At high enough field one minimum disappears.

Mathematically, this corresponds to:

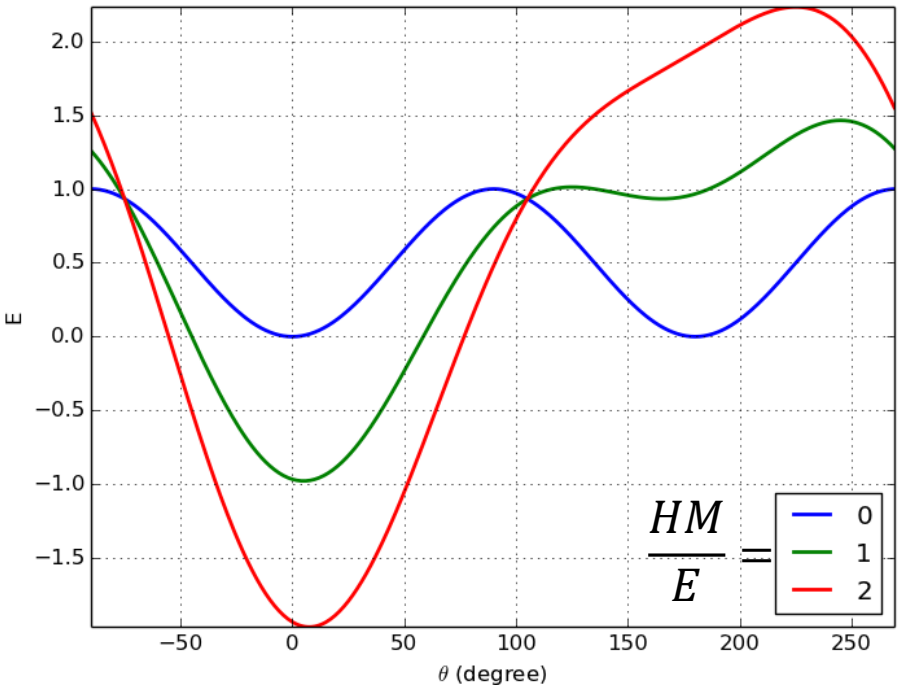
$$\frac{\partial E}{\partial \theta} = 0, \frac{\partial^2 E}{\partial^2 \theta} = 0$$

$$K\sin 2\theta + HM\sin(\phi - \theta) = 0$$
$$2K\cos 2\theta - HM\cos(\phi - \theta) = 0$$

Expand:

$$\frac{2K}{M}\sin\theta\cos\theta + H_x\cos\theta - H_z\sin\theta = 0$$

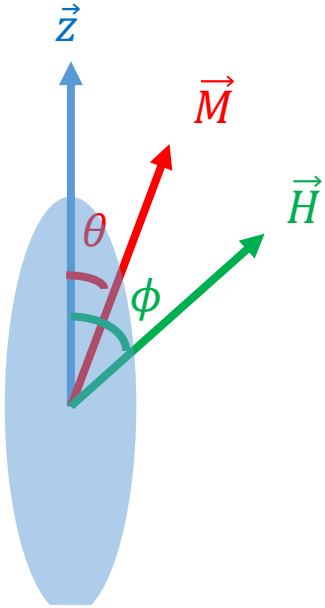
$$\frac{2K}{M}\cos^2\theta - \sin^2\theta - (H_z\cos\theta + H_x\sin\theta) = 0$$



$$H_x = H\sin\theta;\ H_z = H\cos\theta$$

Stoner Wohlfarth model: random field angle example

Total energy: $E = K\sin^2(\theta) + HM\cos(\theta - \phi)$



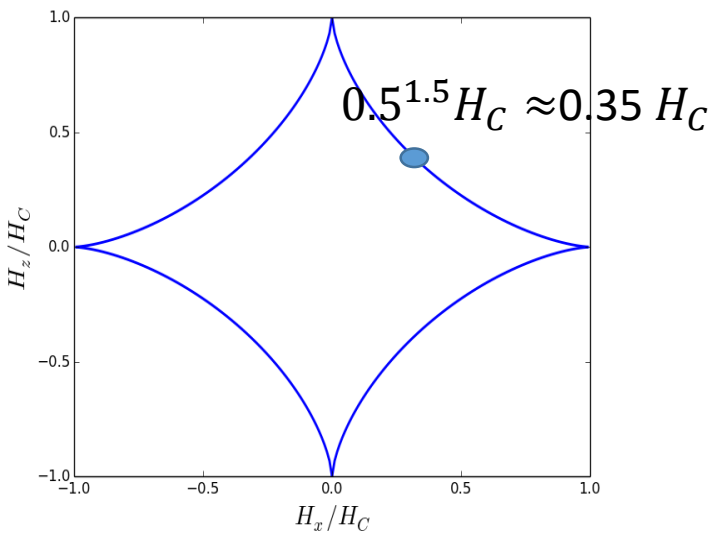
Expand:

$$\frac{2K}{M}\sin\theta\cos\theta + H_x\cos\theta - H_z\sin\theta = 0$$

$$\frac{2K}{M}\cos^2\theta - \sin^2\theta - (H_z\cos\theta + H_x\sin\theta) = 0$$

$$\frac{2K}{M}\sin\theta\cos\theta + H_x\cos\theta - H_z\sin\theta = 0\ [\times\sin\theta]$$

$$\frac{2K}{M}\cos^2\theta - \sin^2\theta - (H_z\cos\theta + H_x\sin\theta) = 0\ [\times\cos\theta]$$



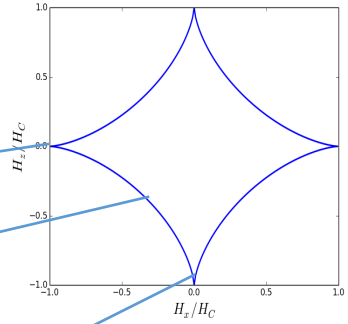
$$H_z = \frac{2K}{M}\cos^3\theta$$

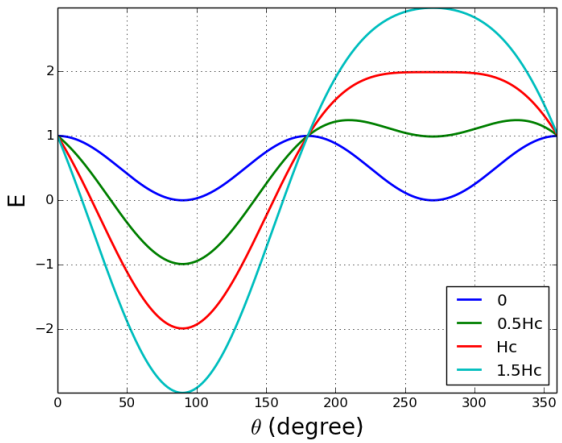
$$H_x = -\frac{2K}{M}\sin^3\theta$$

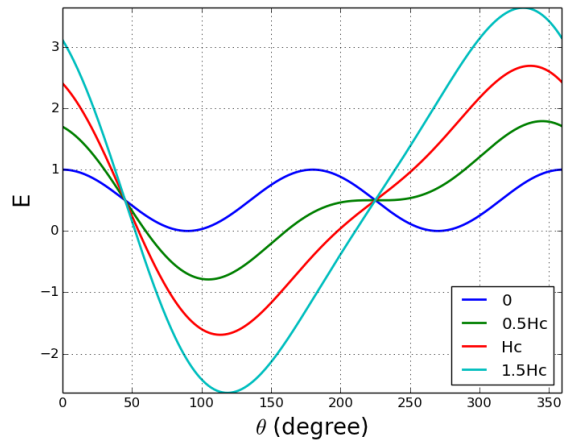
$$H_x^{\frac{2}{3}} + H_z^{\frac{2}{3}} = \left(\frac{2K}{M}\right)^{\frac{2}{3}}$$

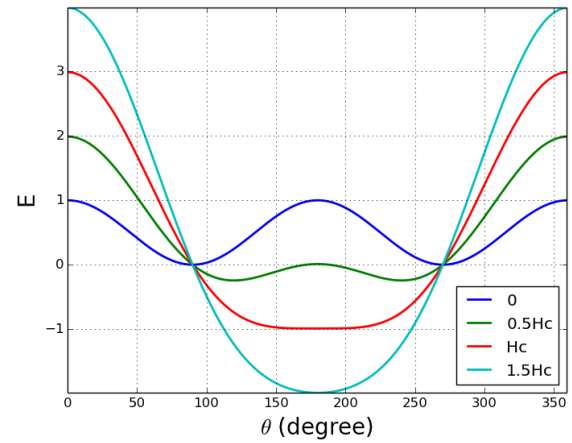
$$H_c = \frac{2K}{M}$$

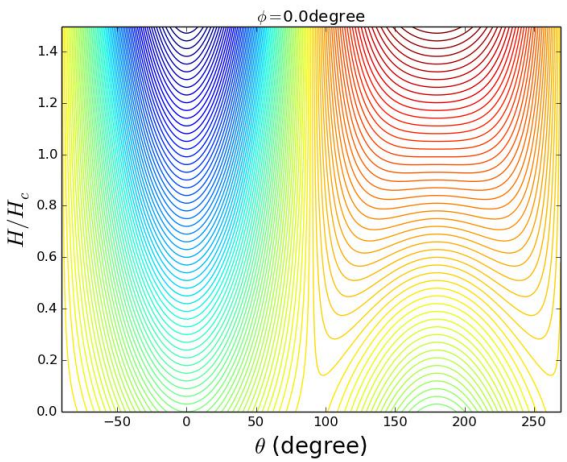
Slonczewski asteroid

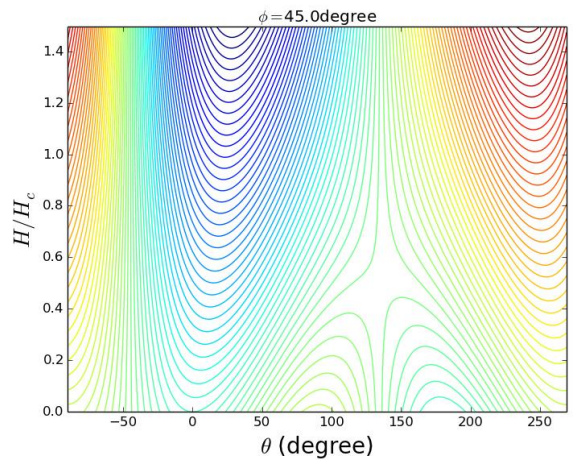


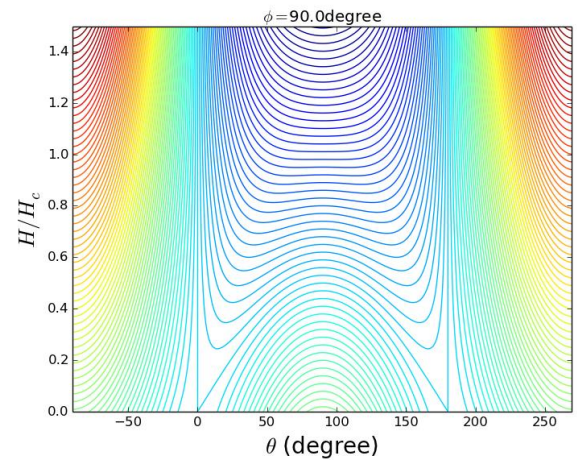




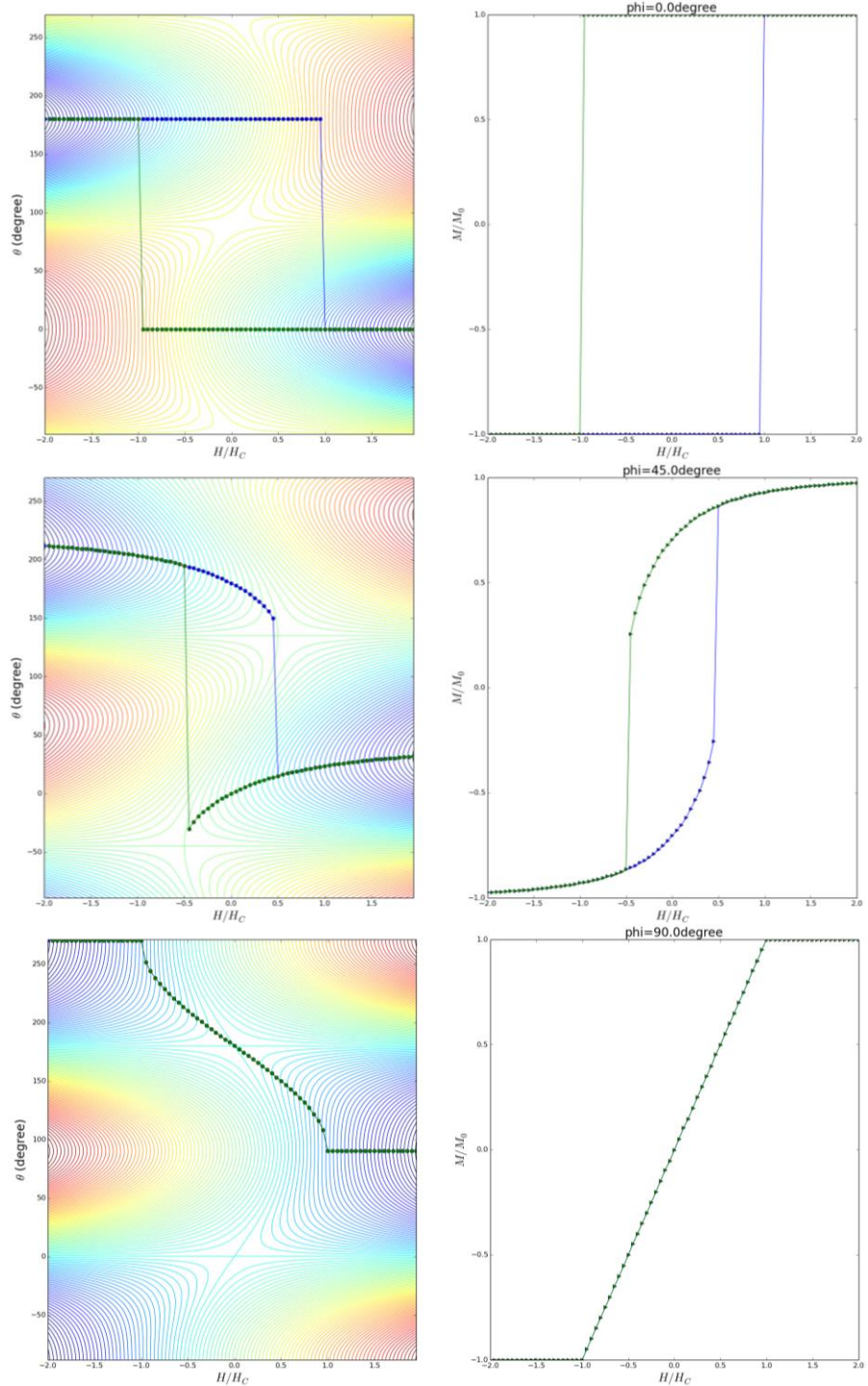




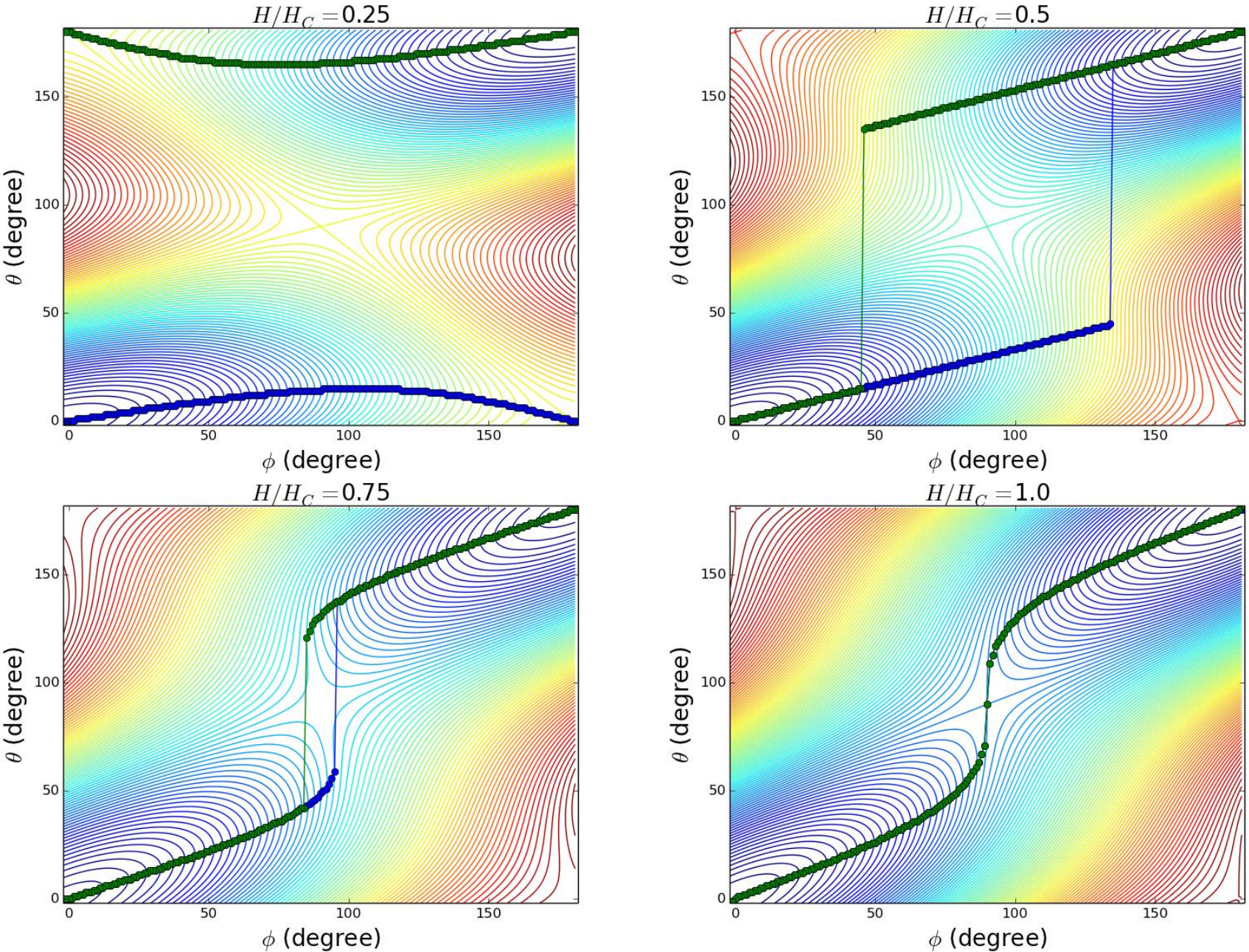




Hysteresis



Angular dependence (anisotropic MR)



Conclusion

- Stoner Wohlfarth model describes the magnetic anisotropy problem of a single domain particle
- It can explain the ideal case hysteresis and angular dependence of magnetizations.