Matrix Analysis of MOKE Effect

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Why Matrix Methods?

- Solutions should always account for boundary conditions
  - One of more important concepts in E&M
- $\vec{E}_{\parallel}$ conserved: ${E_x}^1 = {E_x}^2$ & ${E_y}^1 = {E_y}^2$
- $\vec{H}_{\parallel}$ conserved (no surface current): ${H_x}^1 = {H_x}^2$ & ${H_y}^1 = {H_y}^2$

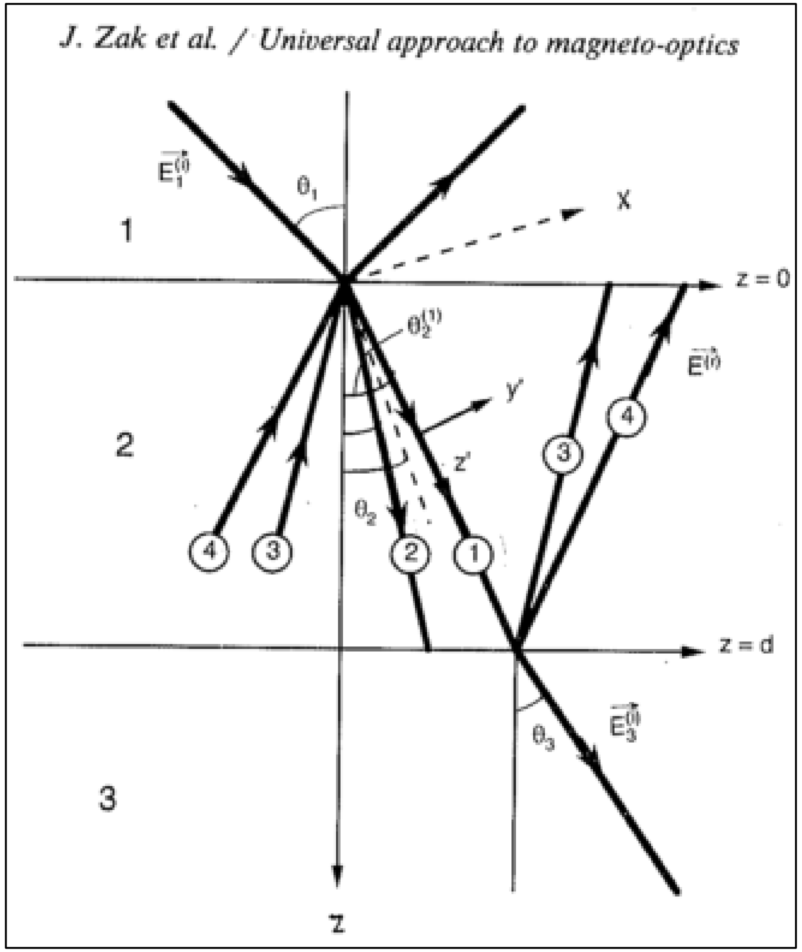
Medium Boundary Matrix

- $F = \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix}$, $P = \begin{pmatrix} {E_s}^{(i)} \\ {E_p}^{(i)} \\ {E_s}^{(r)} \\ {E_p}^{(r)} \end{pmatrix}$ describe the components of the field *incident on* the material (F) and the components *in* the material (P)
- s-polarization: perpendicular to plane of incidence
- p-polarization: parallel to plane of incidence

- Connected by a *medium boundary matrix* A such that F=AP.

Diagram of Interaction

- Incident light comes from material (1)
  - Often (but not required to be) vacuum
- Refracted rays split by angles $\theta_2{}^{(1)}, \theta_2{}^{(2)}$ due to magnetization of material (2)



Refractive Indices by Magneto-optic Constant

- In the two common geometries *polar* and *longitudinal*, the different angles come about from different indices of refraction:
  - ${n_{pol}}^{(1,2)} = N\left(1 \pm \frac{1}{2}\alpha_z Q\right)$
  - ${n_{lon}}^{(1,2)} = N\left(1 \pm \frac{1}{2}\alpha_y Q\right)$
  - $\alpha_z = \cos(\theta_2)$, and $\alpha_y = \sin(\theta_2)$

(Skipping some long derivations here…)
See paper attached to email for details

Medium Boundary Matrix Results

- Get two matrices: one each for polar and longitudinal cases:

$$A^{(\mathrm{POL})} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ \frac{\mathrm{i}}{2}\alpha_y^2 Q & \alpha_z & \frac{\mathrm{i}}{2}\alpha_y^2 Q & -\alpha_z \\ \frac{\mathrm{i}}{2}\alpha_z QN & -N & -\frac{\mathrm{i}}{2}\alpha_z QN & -N \\ \alpha_z N & \frac{\mathrm{i}}{2} QN & -\alpha_z N & \frac{\mathrm{i}}{2} QN \end{pmatrix},$$

$$A^{(\mathrm{LON})} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ -\frac{\mathrm{i}}{2}\frac{\alpha_y}{\alpha_z}(1+\alpha_z^2)Q & \alpha_z & \frac{\mathrm{i}}{2}\frac{\alpha_y}{\alpha_z}(1+\alpha_z^2)Q & -\alpha_z \\ \frac{\mathrm{i}}{2}\alpha_y QN & -N & \frac{\mathrm{i}}{2}\alpha_y QN & -N \\ \alpha_z N & \frac{\mathrm{i}}{2}\frac{\alpha_y}{\alpha_z} QN & -\alpha_z N & -\frac{\mathrm{i}}{2}\frac{\alpha_y}{\alpha_z} QN \end{pmatrix}.$$

- For non-magnetic case, Q=0 simplifies both matrices to the one case:

$$A_1 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & \alpha_{1z} & 0 & -\alpha_{1z} \\ 0 & -N_1 & 0 & -N_1 \\ \alpha_{1z} N_1 & 0 & -\alpha_{1z} N_1 & 0 \end{pmatrix}.$$

How is This Helpful?

- Relate transmission, reflection coefficients to the transmission and reflection coefficients:

$$r_{ss} = \frac{E_{1s}^{(r)}}{E_{1s}^{(i)}}, \quad r_{ps} = \frac{E_{1p}^{(r)}}{E_{1s}^{(i)}}, \quad t_{ss} = \frac{E_{2s}^{(i)}}{E_{1s}^{(i)}}, \quad t_{ps} = \frac{E_{2p}^{(i)}}{E_{1s}^{(i)}}.$$

- r_{ss}: s-polarization light reflecting to s-polarization
- r_{sp}: s-polarization light reflecting to p-polarization
- t_{ss}: s-polarization light transmitting to s-polarization
- t_{sp}: s-polarization light transmitting to p-polarization

How is This Helpful?

- Boundary conditions require that $A_1P_1=A_2P_2$
  - Can then relate to the r, t coefficients
- Can then determine information about Q, which is information about sample magnetization

Extensions of This Method

- Any number of layers can be dealt with
  - e.g. light goes through view window, off of sample, interacts with substrate

- Later work generalizes to arbitrary magnetization (not purely polar and longitudinal)
  - Zak, Moog, Liu and Bader, Phys. Rev. B. **43**, 8 (1991)