Magneto optical Kerr Effect (MOKE)

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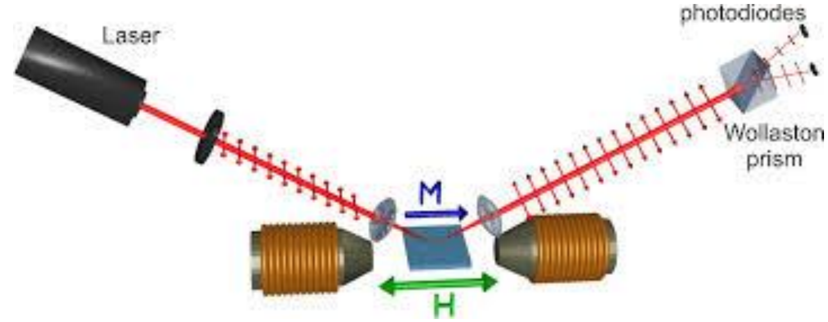
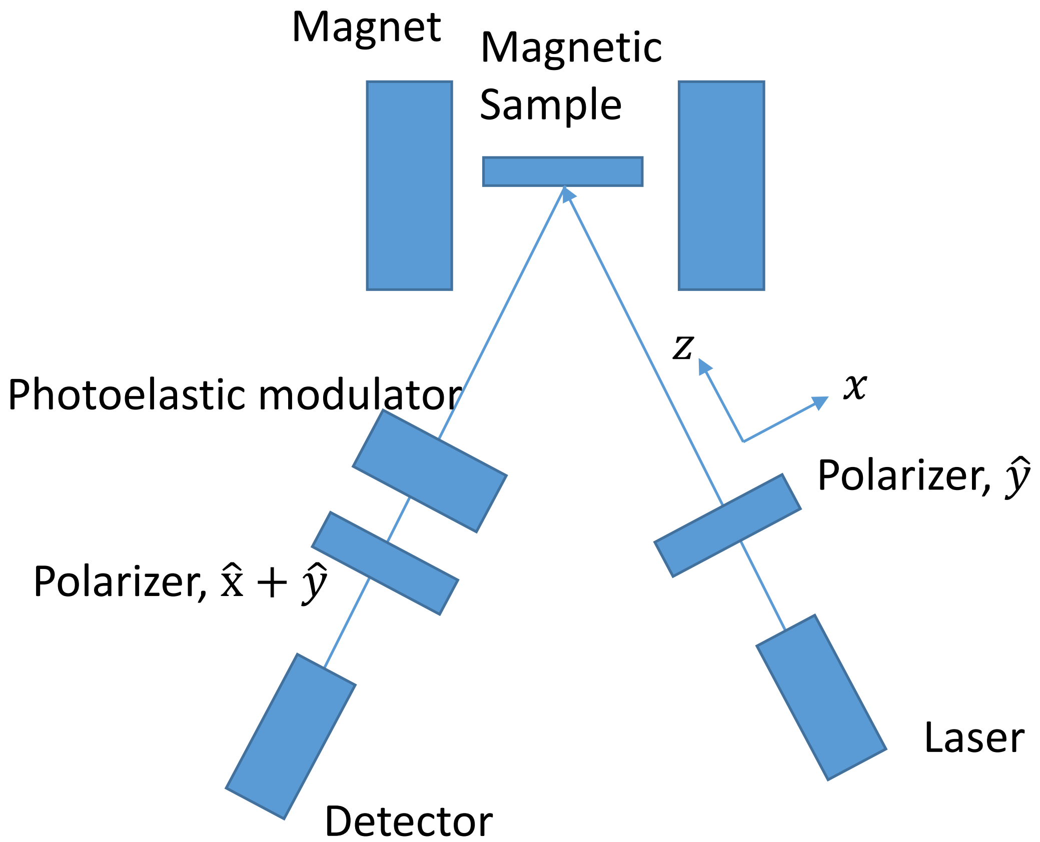
What is MOKE?



- Magneto-optical Kerr Effect
  - The polarization of the light will be rotated after reflection from a magnetic surface, and the rotation depends on the magnetization M of the surface.
- Application
  - Since the magnetization M affects the reflected light, from reflected light, one can derive the magnetization information. (e.g. M-H relation)


Magnet
Magnetic Sample
Photoelastic modulator
Polarizer, $\hat{x} + \hat{y}$
Detector
z
x
Polarizer, $\hat{y}$
Laser

After passing the magnetic material:

Incident light after passing the linear polarizer

$$\frac{\boldsymbol{E}}{E_0 e^{i(\omega t - kz)}} = \hat{\mathrm{y}}$$

After reflection:

$$\frac{\boldsymbol{E}}{E_0 e^{i(\omega t - kz)}} = \beta \hat{\mathrm{x}} + e^{i\gamma} \hat{\mathrm{y}}$$

where β and γ are small numbers (on the order of 1%).

Here β depends on magnetization:

$$\beta = \beta_0 + \beta(M)$$

β_0 represents the general rotation for reflection on any material.

$\beta(M)$ is the part that depends on magnetization, which is normally small (~0.01%)

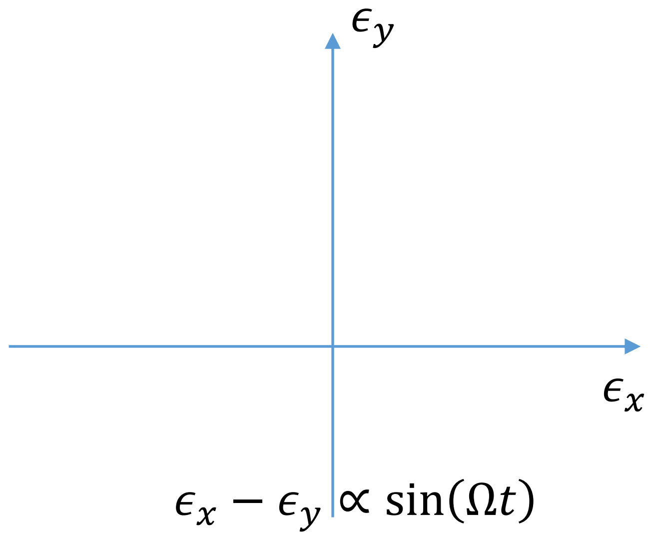
So we are measuring a small change $\beta(M)$ of a small number β in a magnetic field.

After passing the photoelastic modulator (PEM)

In this case, there will be a phase modulation between the $\hat{x}$ and $\hat{y}$ components. Let's assume the modulations is on $\hat{y}$ components by $\theta = A\sin(\Omega t)$.

So the light becomes:

$$\frac{2\boldsymbol{E}}{E_0 e^{i(\omega t - kz)}} = \beta\hat{\mathrm{x}} + e^{i\gamma}\hat{\mathrm{y}}e^{iA\sin(\Omega t)}$$



Measure using a 45 degree polarizer

Passing the 45 degree polarizer:

The light is:

$$\frac{\boldsymbol{E}}{E_0 e^{i(\omega t - kz)}} = \frac{\sqrt{2}}{2}[\beta + e^{i\gamma}e^{i\theta}]$$

$$\frac{\sqrt{2}\boldsymbol{E}}{E_0 e^{i(\omega t - kz)}} = \beta + e^{i\gamma}e^{i\theta}$$

Real part:

$$\beta\cos(\omega t) + \cos(\gamma + \theta + \omega t)$$

Imaginary part:

$$\beta\sin(\omega t) + \sin(\gamma + \theta + \omega t)$$

Intensity:

$$\frac{8I}{E_0^2} = \frac{1}{T}\int [\beta\cos(\omega t) + \cos(\gamma + \theta + \omega t)]^2\, dt$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{1}{T}\int_0^T \cos^2(\gamma + \theta + \omega t)\, dt + \frac{2\beta}{T}\int_0^T \cos(\omega t)\cos(\gamma + \theta + \omega t)\, dt + \frac{\beta^2}{T}\int_0^T \cos^2(\omega t)\, dt$$

Intensity (Note that we can ignore the time dependence of θ, since $\omega \gg \Omega$, 10^{14} and 10^{4}):

$$\frac{1}{T}\int_0^T \cos^2(\gamma+\theta+\omega t)\,dt + \frac{2\beta}{T}\int_0^T \cos(\omega t)\cos(\gamma+\theta)\,dt + \frac{\beta^2}{T}\int_0^T \cos^2(\omega t)\,dt$$

$$\frac{1}{T}\int_0^T \cos^2(\gamma+\theta+\omega t)\,dt = \frac{1}{T}\int_0^T \frac{1+\cos(2\gamma+2\theta+2\omega t)}{2}\,dt = \frac{1}{2} = I_{DC}$$ This is the DC signal

$$\frac{2\beta}{T}\int_0^T \cos(\omega t)\cos(\gamma+\theta)\,dt = \frac{2\beta}{T}\int_0^T \frac{\cos(\gamma+\theta)+\cos(\gamma+\theta+2\omega t)}{2}\,dt$$

$$= \frac{\beta}{T}\int_0^T \cos(\gamma+\theta)\,dt + \frac{\beta}{T}\int_0^T \cos(\gamma+\theta+2\omega t)\,dt$$

$$= \beta\cos(\gamma+\theta) + 0 = \sqrt{2}\mathrm{I}_{\mathrm{AC}}$$

$$\frac{\beta^2}{T}\int_0^T \cos^2(\omega t)\,dt = \frac{\beta^2}{T}\int_0^T \frac{cos(2\omega t)+1}{2}\,dt = \frac{\beta^2}{2}$$

$$\boxed{\frac{I_{AC}}{I_{DC}} \approx \frac{\beta}{\sqrt{2}}}$$

We can actually resolve the time dependence of θ using a oscilloscope.

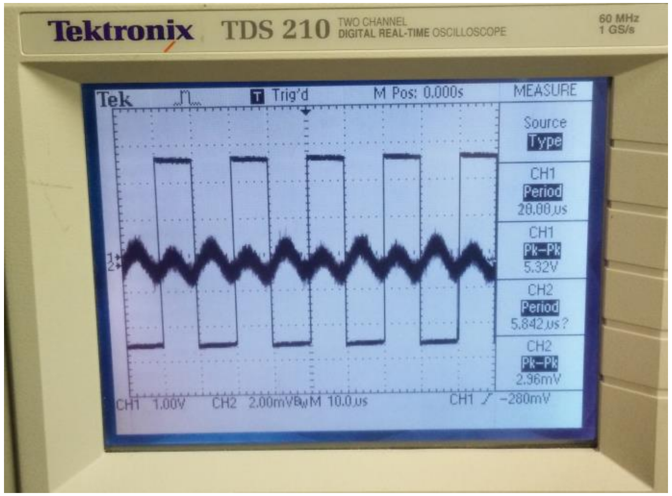
$$\frac{8I}{E_0^2} = \frac{1}{T}\int \left[\beta\cos(\omega t) + \cos(\gamma + \theta + \omega t)\right]^2 dt$$

$$= \frac{1+\beta^2}{2} + \beta\int_0^{\frac{2\pi}{\omega}} \cos(\gamma+\theta)\,dt + \beta^2\int_0^{\frac{2\pi}{\omega}} \cos(2\gamma + 2\theta + 2\omega t)\,dt + \int_0^{\frac{2\pi}{\omega}} \cos(\gamma + \theta + 2\omega \mathrm{t})\,dt$$

If $\omega \gg \Omega$, we can treat θ as a constant

$$\int_0^{\frac{2\pi}{\omega}} \cos(2\gamma + 2\theta + 2\omega t)\,dt \approx 0$$

$$\int_0^{\frac{2\pi}{\omega}} \cos(\gamma + \theta + 2\omega \mathrm{t})\,dt \approx 0$$



We get

$$\frac{8I}{E_0^2} = \frac{1+\beta^2}{2} + \beta\int_0^{\frac{2\pi}{\omega}} \cos(\gamma+\theta)\,dt$$

$$\frac{\boldsymbol{I_{AC}}}{\boldsymbol{I_{DC}}} \approx \frac{\boldsymbol{\beta}}{\sqrt{\mathbf{2}}}$$