



UNIVERSITY OF NEBRASKA-LINCOLN

Physics & Astronomy

# Hall Effect and its Measurements

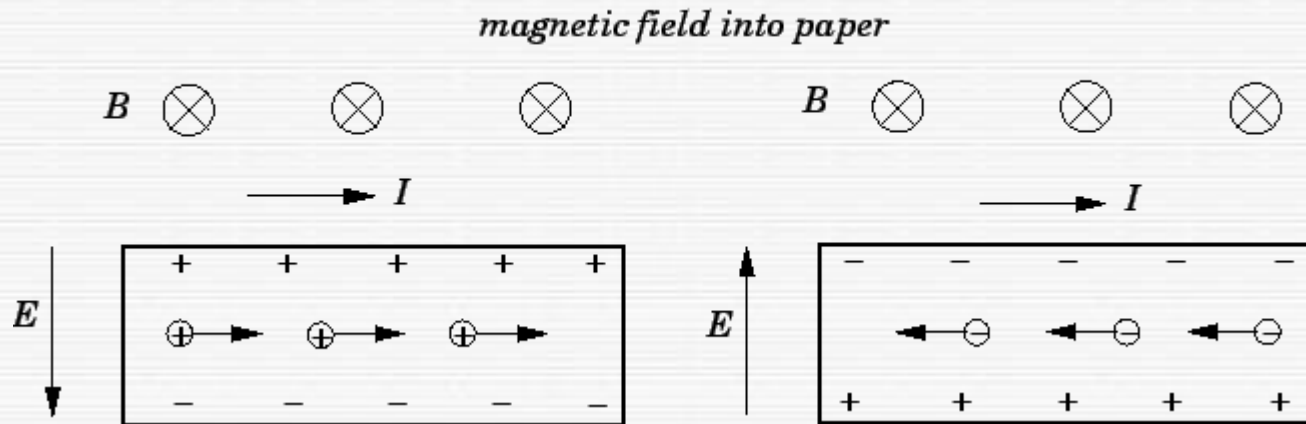
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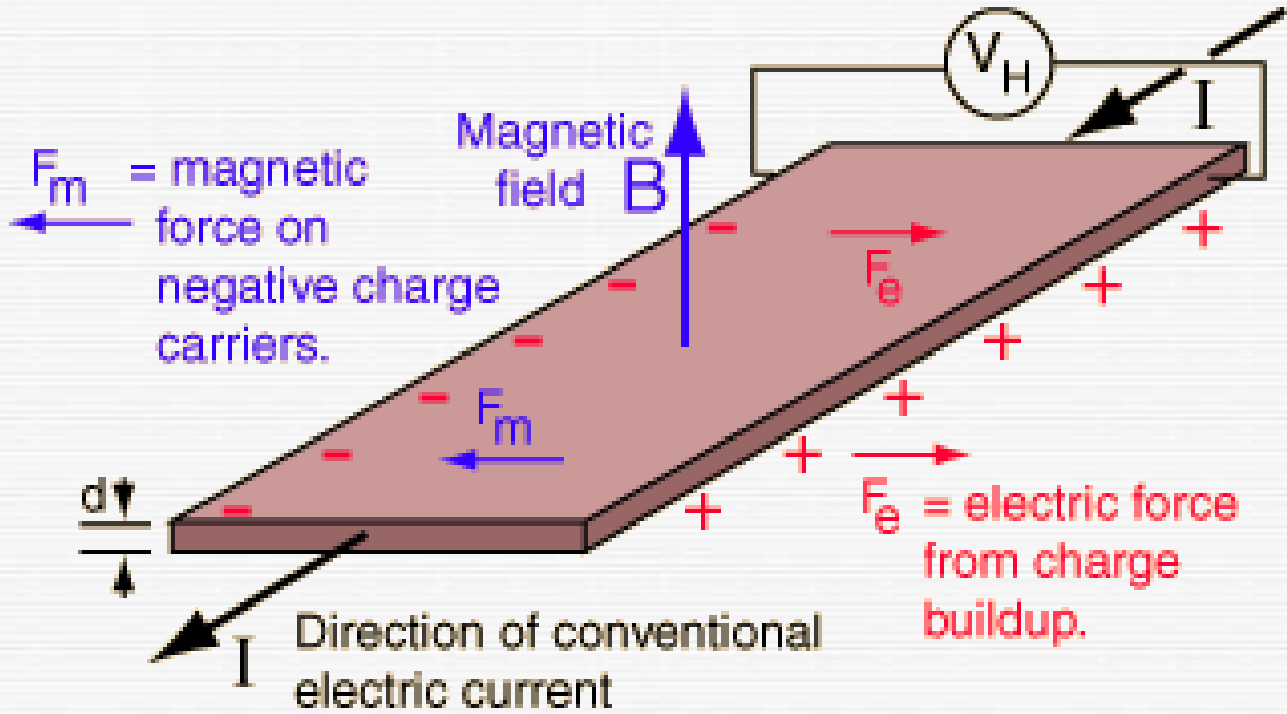
# Hall effect

If an electric current flows through a conductor in a magnetic field, the magnetic field exerts a transverse force on the moving charge carriers which tends to push them to one side of the conductor.





# Hall Voltage for Positive Charge Carriers



$$V_H = \frac{IB}{ned}$$

# Charge Carriers in the Hall Effect



- The Hall effect is **different for different charge carriers**.
- In most common electrical applications, the conventional current is used partly because it makes no difference whether you consider positive or negative charge to be moving.
- The Hall voltage has a different polarity for positive and negative charge carriers, and it has been used to study the details of conduction in semiconductors and other materials which show **a combination of negative and positive charge carriers**.

# Charge Carriers in the Hall Effect

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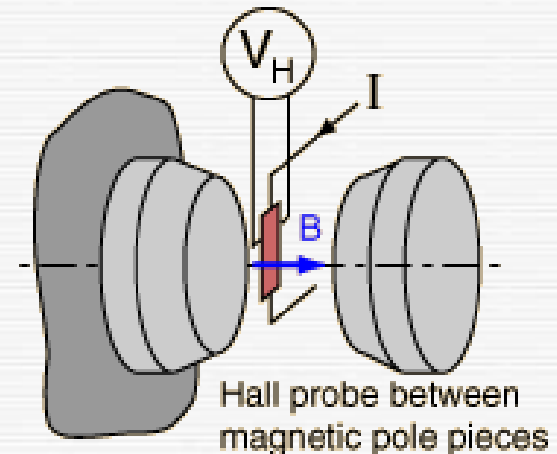
- The Hall effect can be used to measure the **average drift velocity** of the charge carriers
- Mechanically moving the Hall probe at different speeds until the Hall voltage disappears, showing that the charge carriers are now **not moving with respect to the magnetic field**.
- Other types of investigations of carrier behavior are studied in the quantum Hall effect.



# Hall Probe

The measurement of **large magnetic fields on the order of a Tesla** is often done by making use of the Hall effect. A thin film Hall probe is placed in the magnetic field and the transverse voltage (on the order of microvolts) is measured.

Sometimes a thin copper film of thickness  $d$  on the order of 100 micrometers is used for a Hall probe.



The polarity of the Hall voltage for a copper probe shows that electrons are the charge carriers.



# New Discoveries

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- Quantum Hall Effect
- Spin Hall Effect
- Anomalous Hall Effect



# Quantum Hall Effect

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- Quantization of normal Hall Effect
- Seen at **low temperature, high magnetic field**
- Very precise, magnitude determined by Landau levels and electron interaction

$$\sigma = \nu \frac{e^2}{h}$$





# Spin Hall Effect

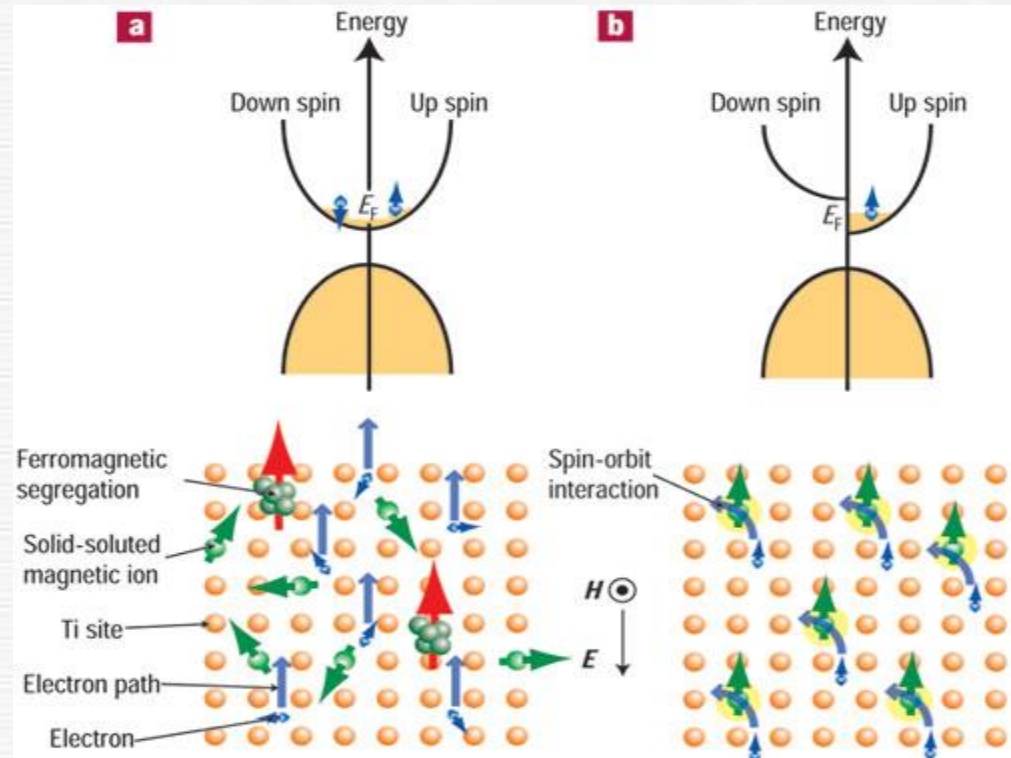
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- Separation of electron spins in current-carrying object, no magnetic field needed
- Predicted in 1971, observed in 2004 via emission of circularly polarized light
- Universal, present in **metals and semiconductors at high and low temperature**

# Anamolous Hall Effect

- Ferromagnetic materials have internal magnetic field

Via Toyosaki et al.  
2004



- Much larger than normal Hall Effect, but not well understood ; Possible Berry-phase effect

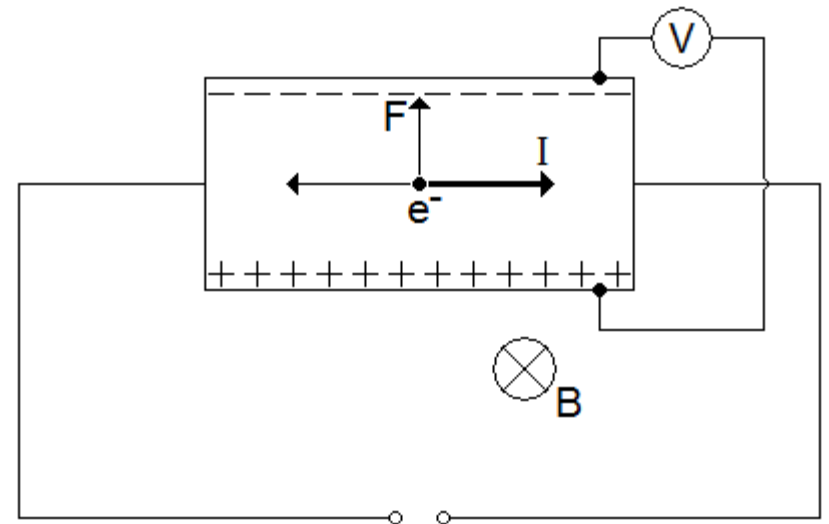


# Hall Effect Measurements

This transverse voltage is the Hall voltage  $V_H$  and its magnitude is equal to  $IB/qnd$ , where  $I$  is the current,  $B$  is the magnetic field,  $d$  is the sample thickness, and  $q$  ( $1.602 \times 10^{-19}$  C) is the elementary charge.

In some cases, it is convenient to use **layer or sheet density ( $n_s = nd$ )** instead of bulk density. One then obtains the equation:

$$n_s = \frac{IB}{q|V_H|}$$





# Hall Effect Measurements

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Thus, by measuring the Hall voltage  $V_H$  and from the known values of  $I$ ,  $B$ , and  $q$ , one can determine the sheet density  $n_s$  of charge carriers in semiconductors. If the measurement apparatus is set up as shown, the Hall voltage is **negative for  $n$ -type semiconductors and positive for  $p$ -type semiconductors**. The sheet resistance  $R_s$  of the semiconductor can be conveniently determined by use of the Van der Paw resistivity measurement technique. Since **sheet resistance involves both sheet density and mobility**, one can determine the Hall mobility from the equation

$$\mu = \frac{|V_H|}{R_s I B} = \frac{1}{q n_s R_s}$$

If the conducting layer thickness  $d$  is known, one can determine the bulk resistivity ( $r = R_s d$ ) and the bulk density ( $n = n_s / d$ ).



# The Van der Pauw Technique

Van der Pauw equation:

$$e^{-\frac{\pi R_A}{R_s}} + e^{-\frac{\pi R_B}{R_s}} = 1$$

In which  $R_A = \frac{V_{43}}{I_{12}}$   $R_B = \frac{V_{14}}{I_{23}}$

The bulk electrical resistivity  $\rho$  can be calculated

$$\rho = R_s d$$

Hall measurement in the Van der Pauw technique is to determine the sheet carrier density  $n_s$  by measuring the Hall voltage  $V_H$

To measure the Hall voltage  $V_H$ ,

current  $I$  contacts 1 and 3

the Hall voltage  $V_H (=V_{24})$  contacts 2 and 4.

the sheet carrier density  $n_s$  can be calculated

$$n_s = \frac{IB}{q|V_H|}$$

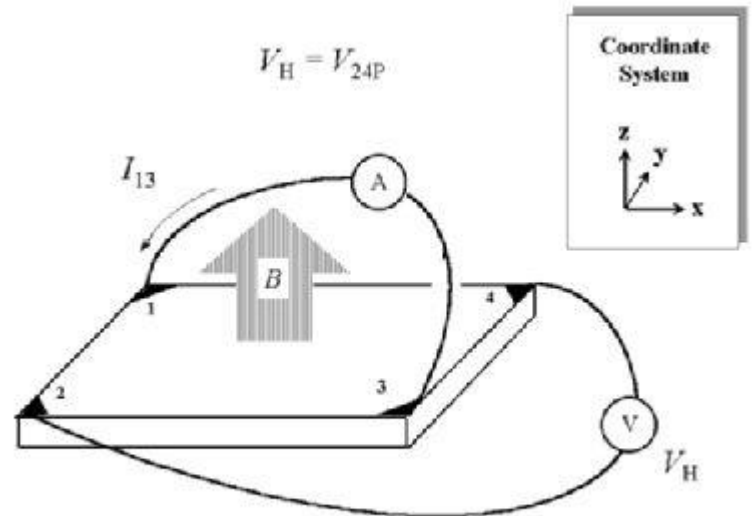
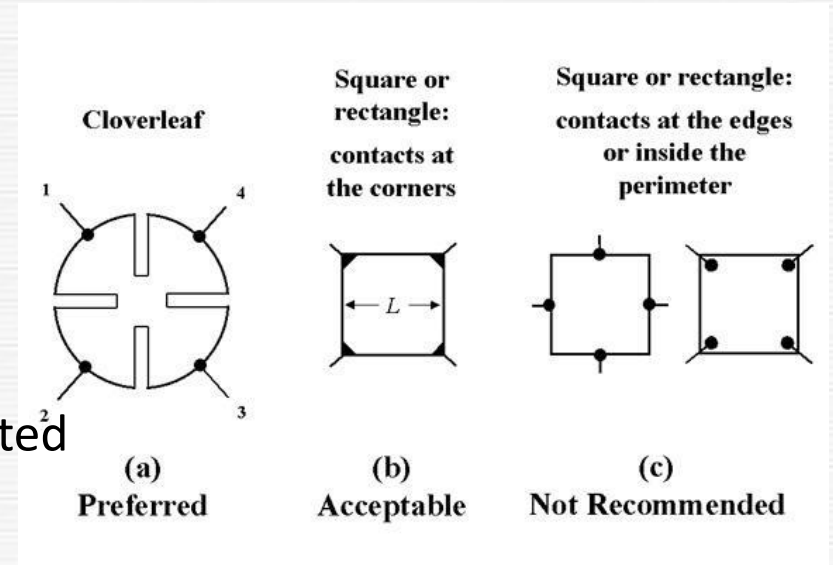


Figure 3



# Summary

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- Hall effect and the new phenomena
- ✓ Quantum Hall Effect
- ✓ Spin Hall Effect
- ✓ Anomalous Hall Effect
- Simple measurement of Hall effect

Next topic: Hall bar, normal Hall effect and Anomalous Hall Effect



**Thank you for your time!**



# Density of Charge Carriers

Calculation of the density of free electrons in a metal like copper involves the basic physical data about the metal, plus the fact that copper provides about one free electron per atom to the electrical conduction process. A representative value can be calculated with the following data.

Molar mass of copper =  $63.54 \text{ gm} / \text{mol} = 63.54 \times 10^{-3} \text{ kg} / \text{mol} = M$

Density of copper =  $9 \text{ gm} / \text{cm}^3 = 9 \times 10^3 \text{ kg} / \text{m}^3 = \rho$

Number of free electrons per mol = Avogadro's number =  $6.02 \times 10^{23} / \text{mol} = N_A$

Number of free electrons per unit volume =  $n$

$$n = \frac{\text{mass} / \text{m}^3 \times \text{atoms} / \text{mol}}{\text{mass} / \text{mol}} = \text{atoms} / \text{m}^3$$

$$n = \frac{\rho N_A}{M} = 8.5 \times 10^{28} \text{ electrons} / \text{m}^3$$

This is a nominal value because the density of copper in electrical wiring cables varies somewhat with processing.