Space Charge Limited Current

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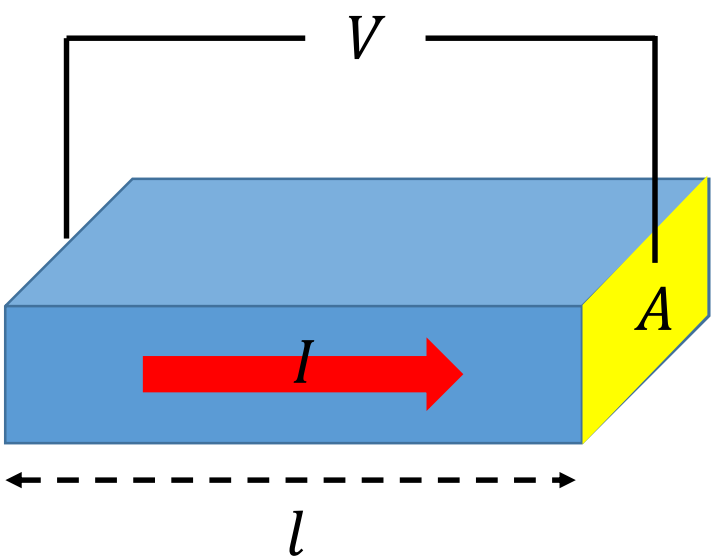
What does electric transport study?

$R = \frac{\rho l}{A}$: resistance

$S = \frac{\sigma A}{l}$: conductance

ρ: resistivity

σ: conductivity



$$j = \sigma E$$

$j = \frac{I}{A}$: current density

$E = \frac{V}{l}$: current density

$$V = R\,I$$
$$I = S\,V$$

Electric transport studies the dependence of σ (or ρ) on electric field (***E***), temperature (***T***), magnetic field (***H***), etc.

Carrier density and mobility

$$j = nv$$

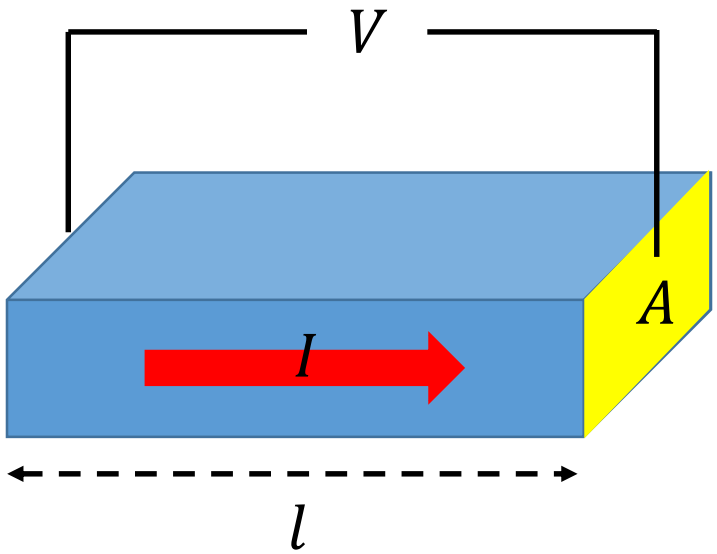
n : carrier density

v : carrier speed

$$j = \sigma E$$

$$\sigma = n\frac{v}{E} = n\mu$$

μ: mobility



$$v = \mu E$$

$$\boxed{j = n\mu E}$$

Why electric transport ?

- Linear electric transport (Ohm's law): ***constant conductivity***
  - Resistive heating
  - Superconductivity
- Non-linear electric transport: ***Electric field-dependent conductivity***
  - P-N junction, diode, transistor
- Thermal resistor: ***temperature-dependent conductivity***
- Magnetoresistance: ***magnetic field-dependent conductivity***

What we really care about?

Conditions (questions)

- Bulk or interface
- Charge neutral or not?
- Crystalline or disorder
- High/low field?
- High/low T

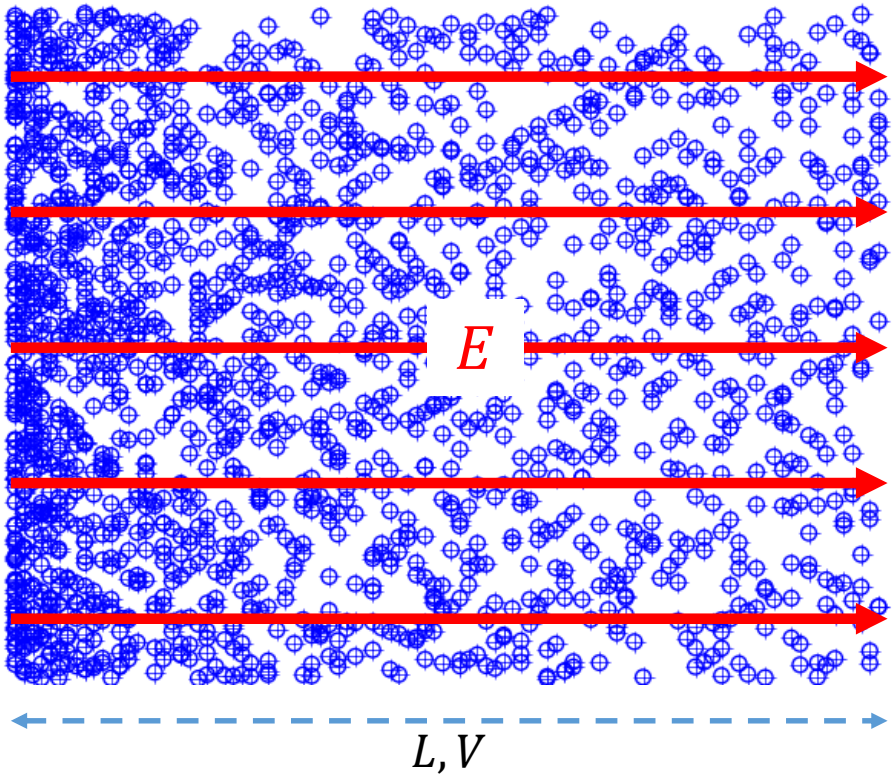
→

Dependences

- Electric field
- Temperature
- Magnetic field

Space charge limited current (SCLC)

- Bulk or interface: bulk
- Charge neutral or not: charged
- Crystalline or disorder: N/A
- High/low field: low
- High/low T: N/A



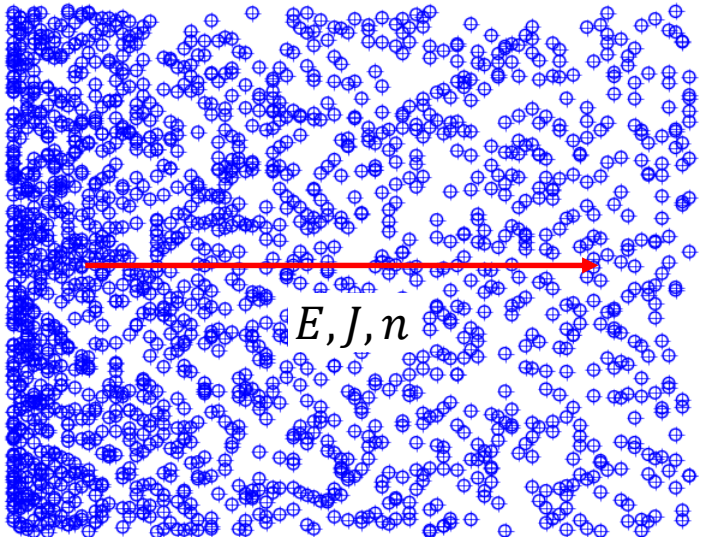
Vacuum or insulator

Mathematical representation

$J = n\mu E$

$\varepsilon \frac{dE}{dx} = n$ Gauss Law

$\mathrm{E} = \frac{dV}{dx}$ Definition of electric field



Solution

We solve $n(x)$ first and then integrate to get $E(x)$ and $V(x)$.

Eliminate E, one gets

$$\varepsilon \frac{d}{dx}\left(\frac{J}{n\mu}\right) = n$$

Or

$$\frac{\varepsilon J}{\mu}\frac{d}{dx}\left(\frac{1}{n}\right) = n$$

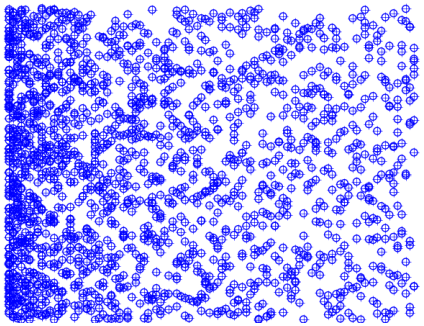
$$\left(-\frac{1}{n^3}\right)\frac{dn}{dx} = \frac{\mu}{\varepsilon J}$$

$$\frac{1}{2}\frac{d}{dx}\left(\frac{1}{n^2}\right) = \frac{\mu}{\varepsilon J}$$

$$\frac{1}{n^2} = \int \frac{2\mu}{\varepsilon J}dx = \frac{2\mu x}{\varepsilon J} \quad assuming\ n = 0\ at\ x = 0$$

One gets:

$$\boxed{n = \sqrt{\frac{\varepsilon J}{2\mu x}}}$$



Solution

$$E = \frac{1}{\varepsilon}\int n dx = \frac{1}{\varepsilon}\int \sqrt{\frac{\varepsilon J}{2\mu x}}\, dx = \sqrt{\frac{J}{2\varepsilon\mu}}\int \sqrt{\frac{1}{x}}\, dx = 2\sqrt{\frac{J}{2\varepsilon\mu}}\, x^{\frac{1}{2}}$$

$$V = \int E dx = \int 2\sqrt{\frac{J}{2\varepsilon\mu}}\, x^{\frac{1}{2}}\, dx = 2\sqrt{\frac{J}{2\varepsilon\mu}}\int x^{\frac{1}{2}}\, dx = 2\,\frac{2}{3}\sqrt{\frac{J}{2\varepsilon\mu}}\, x^{\frac{3}{2}}$$

$$= \frac{4}{3}\sqrt{\frac{J}{2\varepsilon\mu}}\, x^{\frac{3}{2}}$$

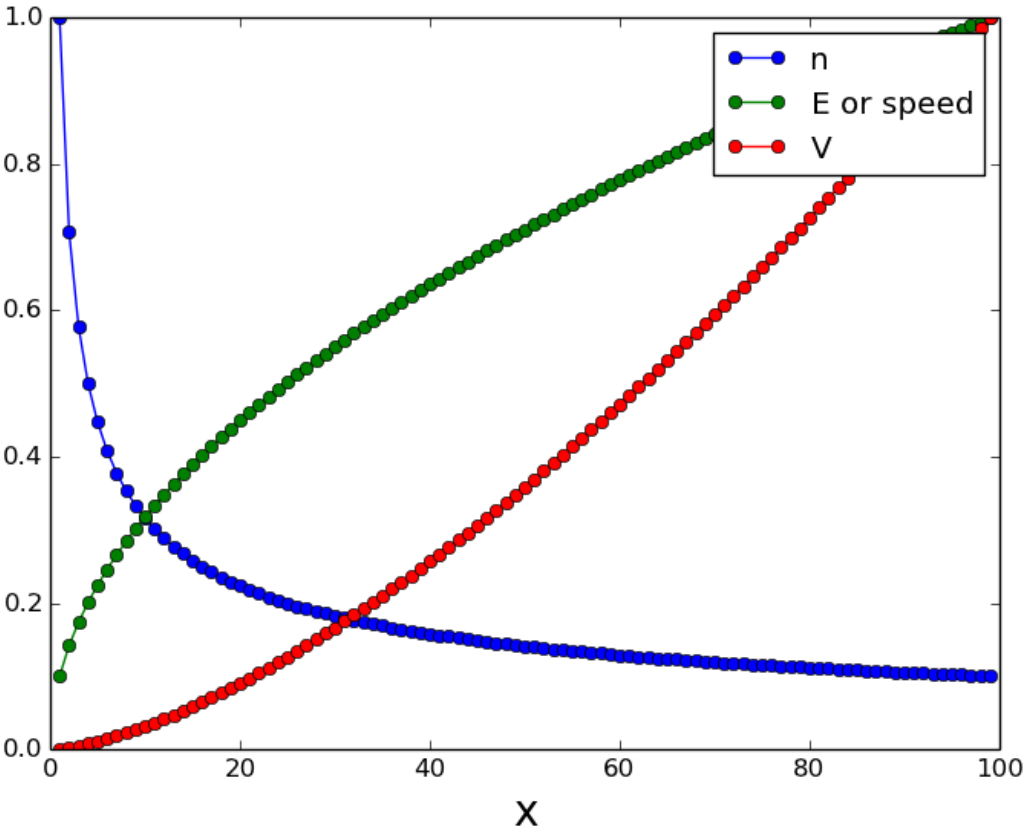
Hence

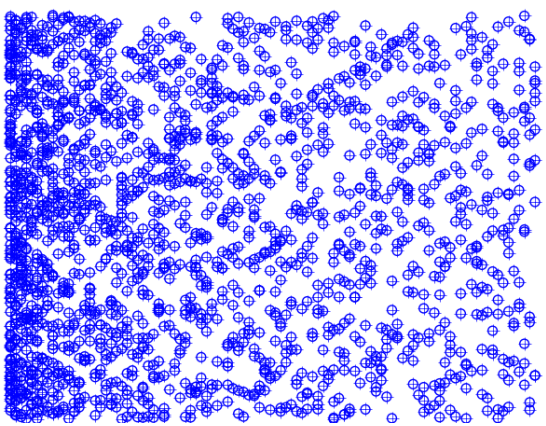
$$V^2 = \frac{16}{9}\frac{J}{2\varepsilon\mu}x^3$$

Or

$$\boxed{J = \frac{9}{8}\varepsilon\mu\frac{V^2}{x^3}}$$

Solution

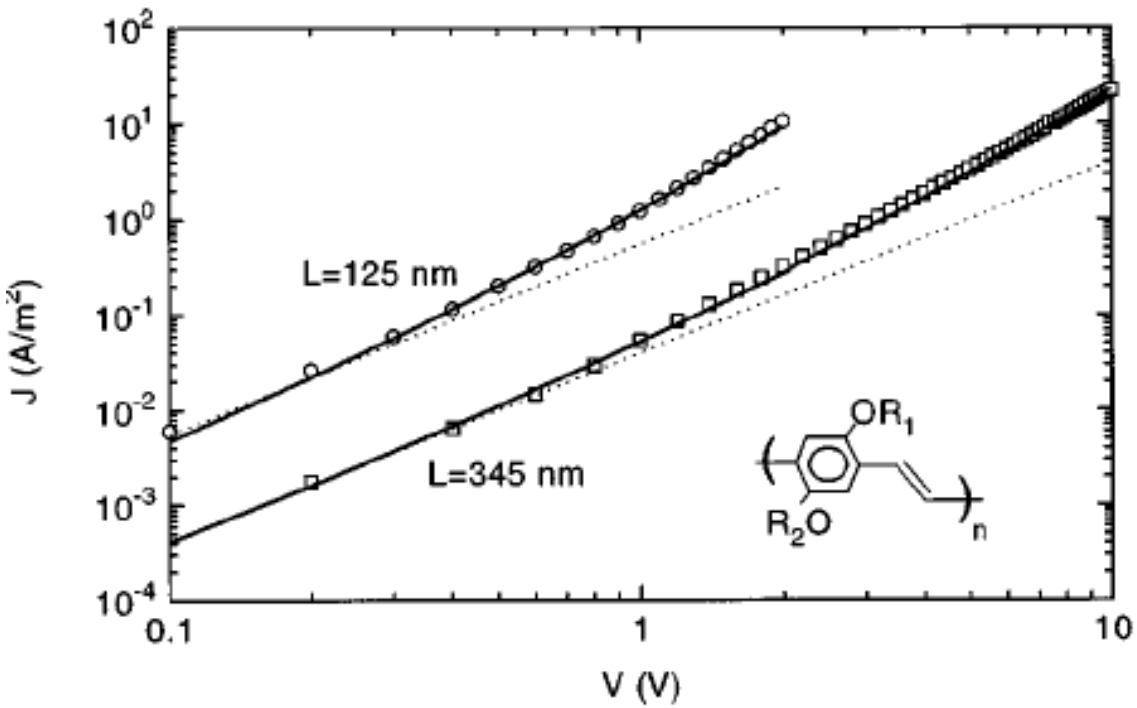




x

$$J = \frac{9}{8}\varepsilon\mu\frac{V^2}{x^3}$$

Application



PRB, v55, R656, (1997).

Electric-field and temperature dependence of the hole mobility in poly.*p*-phenylene vinylene.

Conclusion

- Condition:
  - Bulk or interface: bulk
  - Charge neutral or not: charged
  - Crystalline or disorder: N/A
  - High/low field: low
  - High/low T: N/A

- dependence
  - Electric field: $J = \frac{9}{8}\varepsilon\mu\frac{V^2}{x^3}$
  - Temperature, N/A
  - Magnetic field: N/A