Quantum Tunneling Through a Barrier

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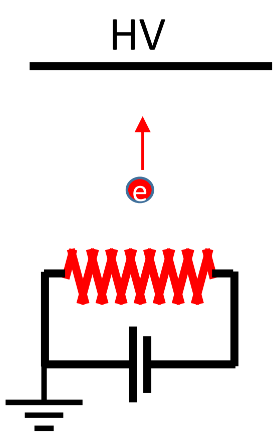
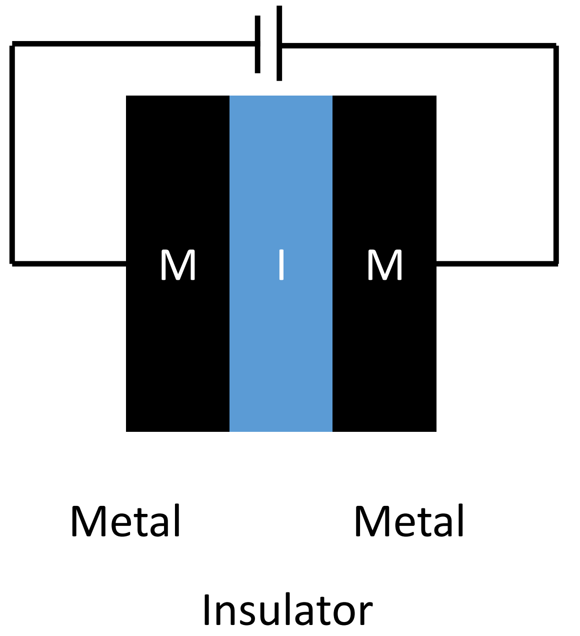
Example of quantum tunneling of electrons

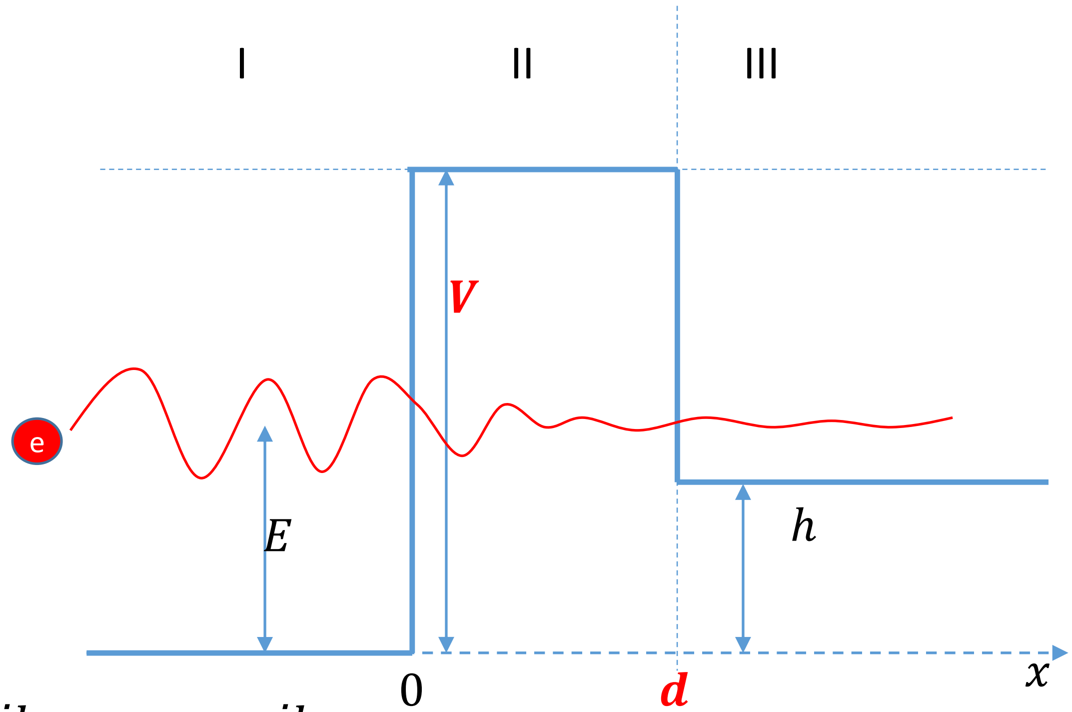
Field emission



Electron transport



1. Simple model of quantum tunneling



$$\psi_I = e^{ikx} + Re^{-ikx}$$

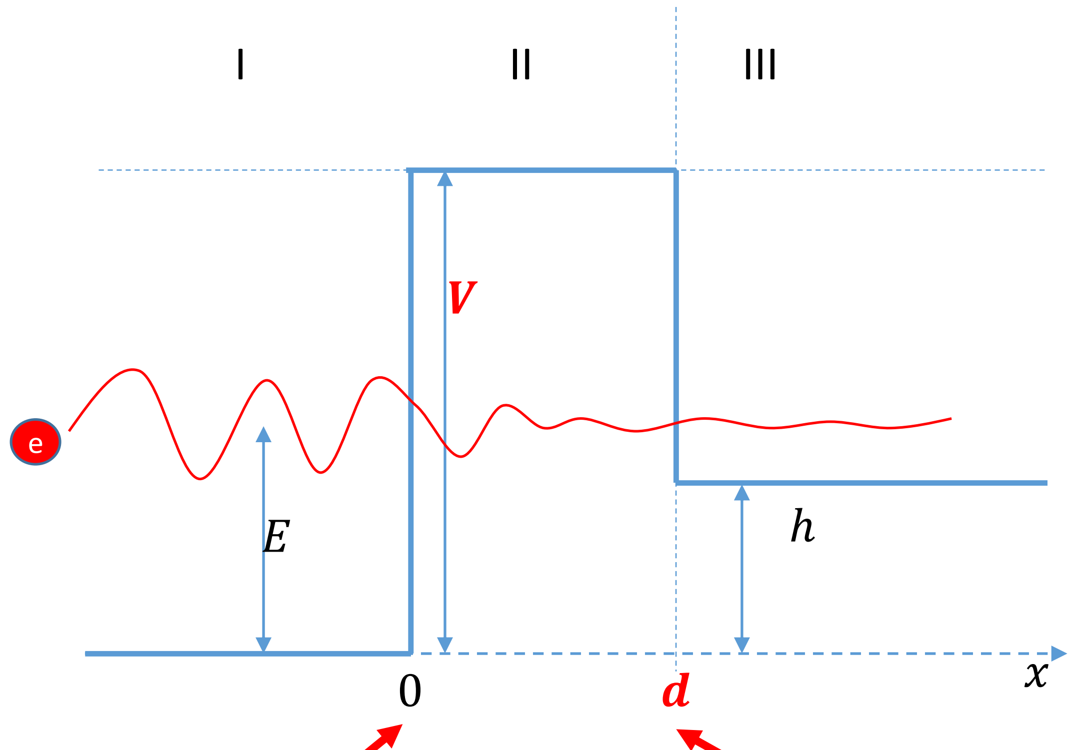
$$\psi_{III} = te^{ik''x}$$

$$\psi_{II} = Ae^{k'x} + Be^{-k'x}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}, k' = \sqrt{\frac{2m(V - E)}{\hbar^2}}, k'' = \sqrt{\frac{2m(E - h)}{\hbar^2}}$$

Boundary conditions to solve the problem

$$k = \sqrt{\frac{2mE}{\hbar^2}}, k' = \sqrt{\frac{2m(V - E)}{\hbar^2}}, k'' = \sqrt{\frac{2m(E - h)}{\hbar^2}}$$

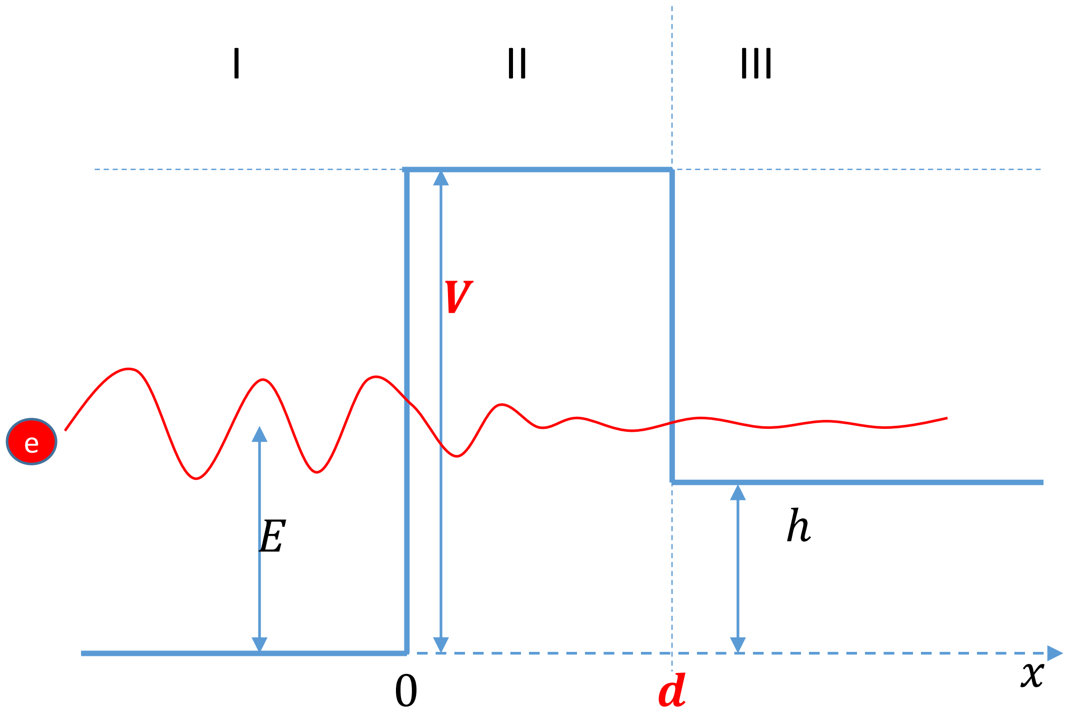


$$\psi_I(0) = \psi_{II}(0)$$
$$\left.\frac{d\psi_I}{dx}\right|_{x=0} = \left.\frac{d\psi_{II}}{dx}\right|_{x=0}$$

$$\psi_{II}(d) = \psi_{III}(d)$$
$$\left.\frac{d\psi_{II}}{dx}\right|_{x=d} = \left.\frac{d\psi_{III}}{dx}\right|_{x=d}$$

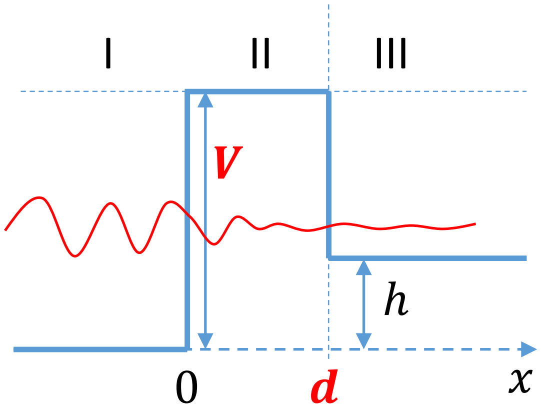
Tunneling probability

$$k = \sqrt{\frac{2mE}{\hbar^2}}, \mathbf{k'} = \sqrt{\frac{2m(\mathbf{V} - E)}{\hbar^2}}, k'' = \sqrt{\frac{2m(E - h)}{\hbar^2}}$$



$$T = |t|^2 = \frac{4(k\mathbf{k'})^2}{\mathbf{k'^2}(k + k'')^2 \cosh^2(\mathbf{k'}d) + (kk'' - \mathbf{k'^2})^2 \sinh^2(\mathbf{k'}d)}$$

Tunneling probability



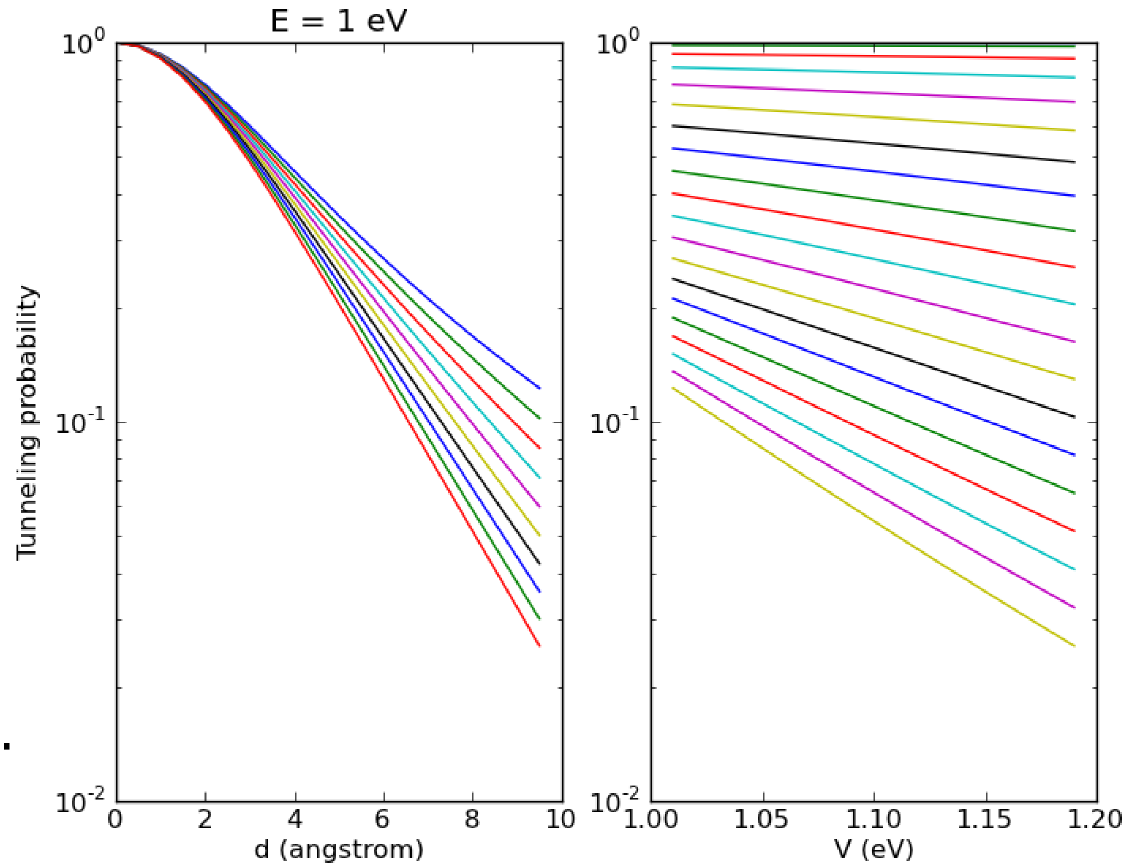
$$k = \sqrt{\frac{2mE}{\hbar^2}}, \quad \boldsymbol{k'} = \sqrt{\frac{2m(\boldsymbol{V} - E)}{\hbar^2}}, \quad k'' = \sqrt{\frac{2m(E - h)}{\hbar^2}}$$

- $k'd \ll 1$

$$T = \frac{4k^2}{(k + k'')^2}\left[1 - \frac{(kk'' - k'^2)^2 d^2}{(k + k'')^2}\right]$$

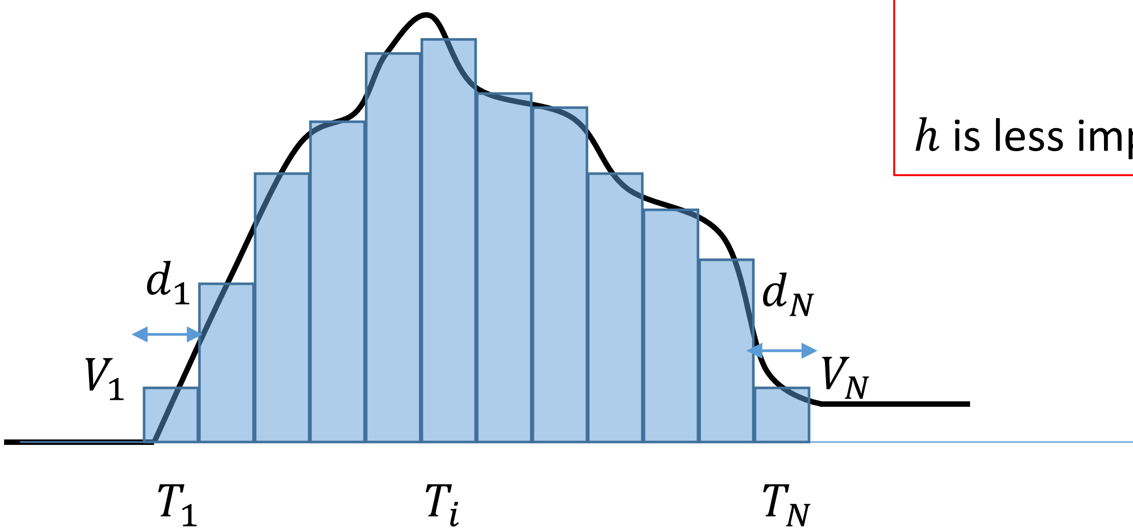
- $k'd \gg 1$

$$T = \frac{4E(V - E)}{V(V - h)} e^{-2\boldsymbol{k'd}}.$$



d ***V***

2. Barriers of arbitrary shape



- $k'd \gg 1$

$$T = \frac{4E(V - E)}{V(V - h)} e^{-2\boldsymbol{k'd}}$$

h is less important than the factor $e^{-2\boldsymbol{k'd}}$

1. Ignore the complexity of h parameter: $T \approx \frac{4E(V-E)}{V^2} e^{-2\boldsymbol{k'd}}$
2. WKB (Wentzel, Kramers, Brillouin) approximation:

$$T = \prod_i T_i = \prod_i \frac{4E(V_i - E)}{V_i^2} e^{-2\boldsymbol{k_i' d_i}} = e^{-2\sum_i \boldsymbol{k_i' d_i}} \prod_i \frac{4E(V_i - E)}{V_i^2}$$

$$\approx e^{-2\int k' dx} \prod_i \frac{4E(V_i - E)}{V_i^2}$$

Fowler-Nordheim Tunneling

Electric field

$V(x) = V_0 - \varepsilon x$

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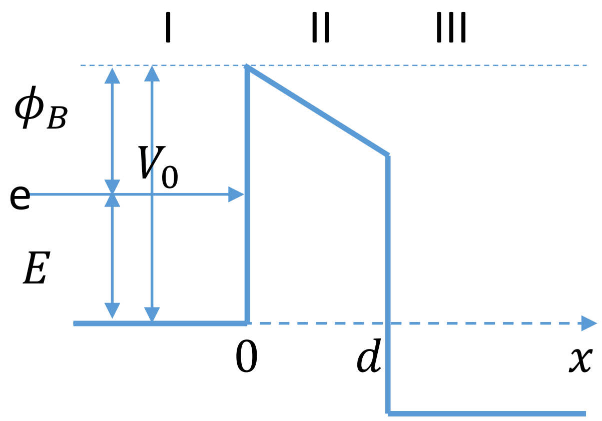
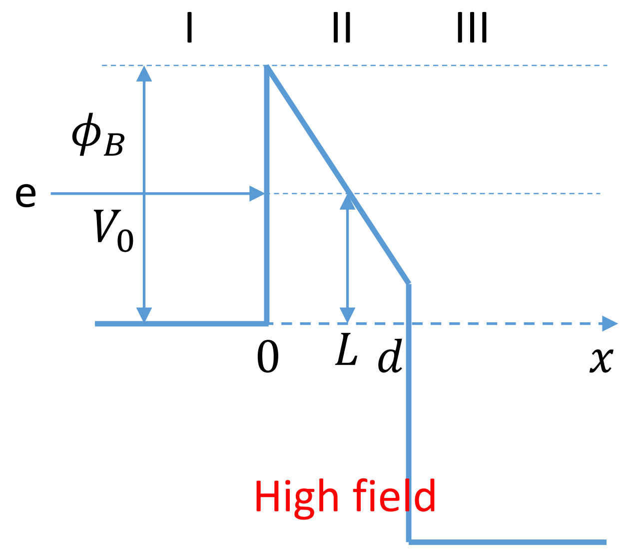


$V_0 = \phi_B + E$

Low field



High field

Electric current: $I \propto T \propto e^{-2\int k' dx} = e^{-\frac{2}{\hbar}\int \sqrt{2m(V-E)}dx}$

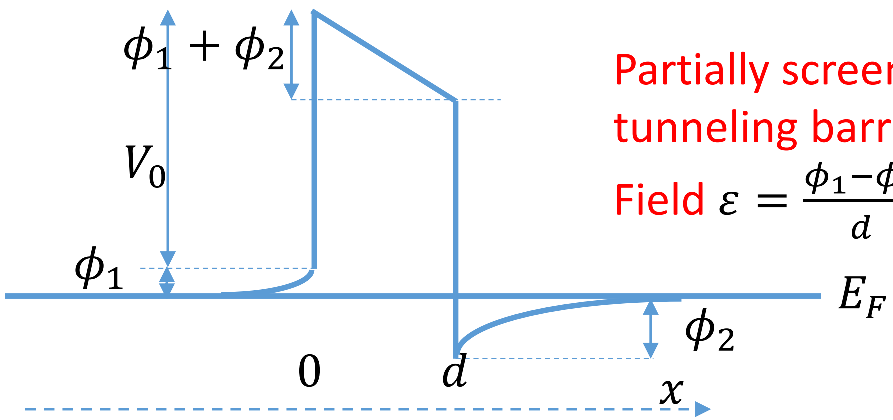
$$I \propto e^{-\frac{4\sqrt{2me}}{3\hbar}\frac{[\phi_B^{\frac{3}{2}}-(\phi_B-\varepsilon L)^{\frac{3}{2}}]}{\varepsilon}}$$

For high field, the tunneling width is L and$\phi_B - \varepsilon L = 0.$

$$I \propto e^{-\frac{4\sqrt{2me}}{3\hbar}\frac{\phi_B^{\frac{3}{2}}}{\varepsilon}}$$

Tunneling through an asymmetric barrier

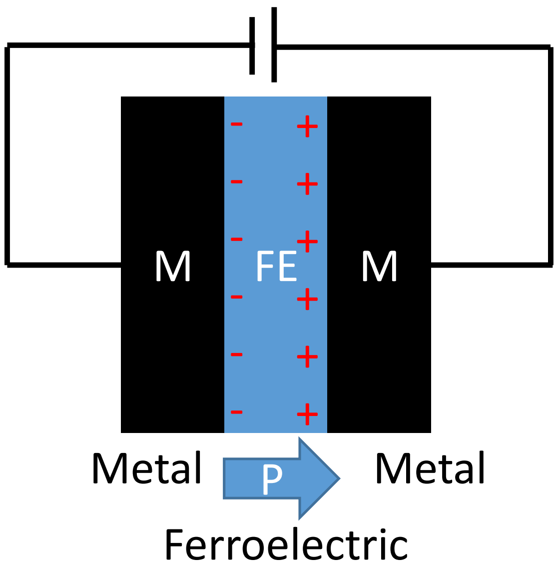
$$V = V_0 + \phi_1 - \frac{\phi_1 + \phi_2}{d} x$$



Partially screened ferroelectric tunneling barrier:

Field $\varepsilon = \frac{\phi_1 - \phi_2}{d}$

Electron transport



- Polarization $\rightarrow$: $V = V_0 + \phi_1 - \frac{\phi_1+\phi_2}{d} x$

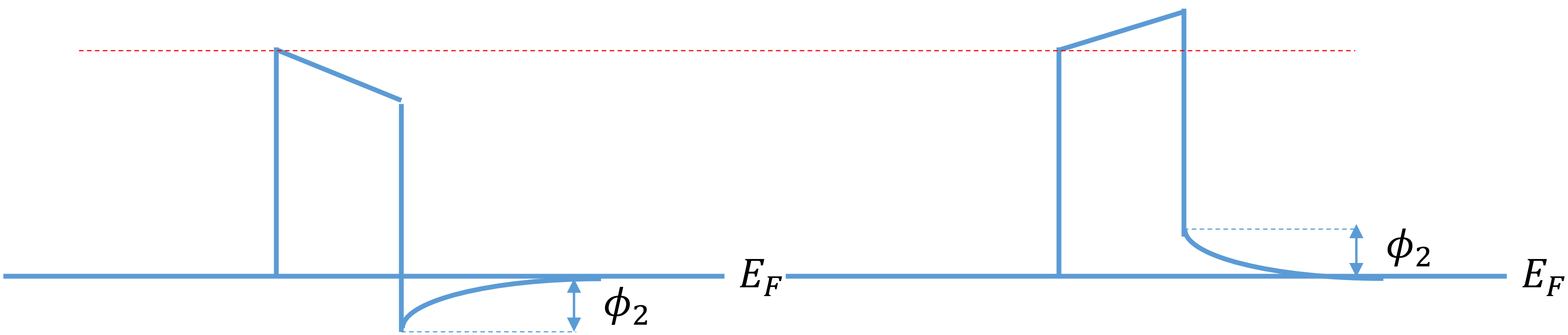
$$I_{\rightarrow} \propto e^{-\frac{4\sqrt{2me}}{3\hbar} \frac{\left[(V_0+\phi_1-E)^{\frac{3}{2}} - (V_0-\phi_2-E)^{\frac{3}{2}}\right]}{\frac{\phi_1+\phi_2}{d}}}$$

- Polarization $\leftarrow$: $V = V_0 - \phi_1 + \frac{\phi_1+\phi_2}{d} x$

$$I_{\leftarrow} \propto e^{-\frac{4\sqrt{2me}}{3\hbar} \frac{\left[(V_0-\phi_1-E)^{\frac{3}{2}} - (V_0+\phi_2-E)^{\frac{3}{2}}\right]}{\frac{\phi_2+\phi_1}{d}}}$$

If $\phi_1 \gg \phi_2$,

Tunneling through an asymmetric barrier



Simplified case: $\phi_1 = 0$,

$$I_{\rightarrow} \propto e^{-\frac{4\sqrt{2me}}{3\hbar}\frac{\left[(V_0-E)^{\frac{3}{2}}-(V_0-\phi_2-E)^{\frac{3}{2}}\right]}{\frac{\phi_2}{d}}}$$

$$I_{\leftarrow} \propto e^{-\frac{4\sqrt{2me}}{3\hbar}\frac{\left[(V_0-E)^{\frac{3}{2}}-(V_0+\phi_2-E)^{\frac{3}{2}}\right]}{\frac{\phi_2}{d}}}$$

$$I_{\leftarrow} > I_{\rightarrow}$$

It is the over all change of the energy landscape :

$$\int \sqrt{2m(V-E)}dx$$

That caused current difference.

Conclusion

- We used a simple quantum mechanical model to explain the electron tunneling
- The simple model can be used to explain:
    - *Fowler-Nordheim Tunneling*
    - *Tunneling through an asymmetric barrier (ferroelectric barrier)*