Matrix Analysis of MOKE Effect

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Why Matrix Methods?

- Solutions should always account for boundary conditions
  - One of more important concepts in E&M

- $\vec{\mathrm{E}}_{\parallel}$ continuous: $\mathrm{E_x}^1 = \mathrm{E_x}^2$ & $\mathrm{E_y}^1 = \mathrm{E_y}^2$

- $\vec{\mathrm{H}}_{\parallel}$ continuous (no surface current): $\mathrm{H_x}^1 = \mathrm{H_x}^2$ & $\mathrm{H_y}^1 = \mathrm{H_y}^2$

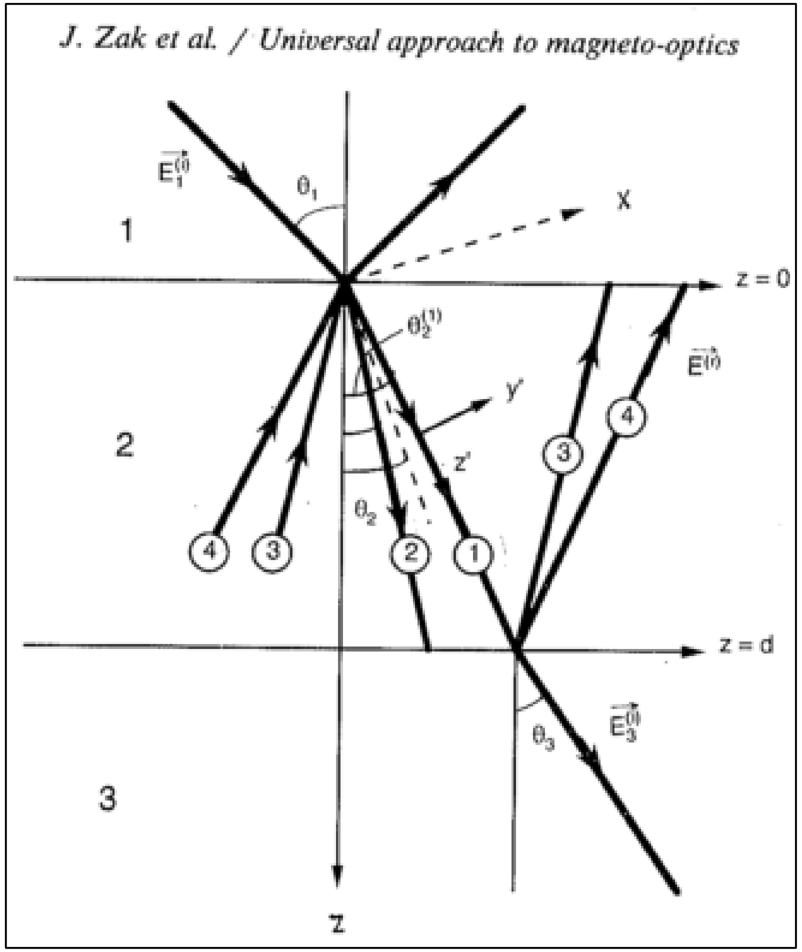
Medium Boundary Matrix

- $F = \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix}, P = \begin{pmatrix} {E_s}^{(i)} \\ {E_p}^{(i)} \\ {E_s}^{(r)} \\ {E_p}^{(r)} \end{pmatrix}$ describe the components of the field *incident on* the material (F) and the components *in* the material (P)
  - e.g. ${E_{s,p}}^{(r)} = 0$ in the two media case
- s-polarization: perpendicular to plane of incidence
- p-polarization: parallel to plane of incidence

- Connected by a *medium boundary matrix* A such that $F = AP$.

Diagram of Three-Media Interaction

- Incident light comes from material (1)
  - Often (but not required to be) vacuum
- Refracted rays split by angles ${\theta_2}^{(1)}, {\theta_2}^{(2)}$ due to magnetization of material (2)
- Rays can reflect off interface at z=d, or penetrate into material (3)



Refractive Indices by Magneto-optic Constant

- In the two common geometries *polar* and *longitudinal*, the different angles come about from different indices of refraction:
  - $n_{pol}{}^{(1,2)} = N\left(1 \pm \frac{1}{2}\alpha_z Q\right)$
  - $n_{lon}{}^{(1,2)} = N\left(1 \pm \frac{1}{2}\alpha_y Q\right)$
  - $\alpha_z = \cos(\theta_2)$, and $\alpha_y = \sin(\theta_2)$

- We can use these to rewrite y, z components of E in terms of x component and α_i's.

Medium Boundary Matrix Results

- Get two matrices: one each for polar and longitudinal cases:

$$A^{(\mathrm{POL})} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ \frac{i}{2}\alpha_y^2 Q & \alpha_z & \frac{i}{2}\alpha_y^2 Q & -\alpha_z \\ \frac{i}{2}\alpha_z QN & -N & -\frac{i}{2}\alpha_z QN & -N \\ \alpha_z N & \frac{i}{2}QN & -\alpha_z N & \frac{i}{2}QN \end{pmatrix},$$

$$A^{(\mathrm{LON})} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ -\frac{i}{2}\frac{\alpha_y}{\alpha_z}(1+\alpha_z^2)Q & \alpha_z & \frac{i}{2}\frac{\alpha_y}{\alpha_z}(1+\alpha_z^2)Q & -\alpha_z \\ \frac{i}{2}\alpha_y QN & -N & \frac{i}{2}\alpha_y QN & -N \\ \alpha_z N & \frac{i}{2}\frac{\alpha_y}{\alpha_z}QN & -\alpha_z N & -\frac{i}{2}\frac{\alpha_y}{\alpha_z}QN \end{pmatrix}.$$

- For non-magnetic case, Q=0 simplifies both matrices to the one case:

$$A_1 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & \alpha_{1z} & 0 & -\alpha_{1z} \\ 0 & -N_1 & 0 & -N_1 \\ \alpha_{1z}N_1 & 0 & -\alpha_{1z}N_1 & 0 \end{pmatrix}.$$

- α_{1z}: cosine of angle to normal in medium (1)

How is This Helpful?

- Relate transmission, reflection coefficients to the transmission and reflection coefficients:

$$r_{ss} = \frac{E_{1s}^{(r)}}{E_{1s}^{(i)}}, \quad r_{ps} = \frac{E_{1p}^{(r)}}{E_{1s}^{(i)}}, \quad t_{ss} = \frac{E_{2s}^{(i)}}{E_{1s}^{(i)}}, \quad t_{ps} = \frac{E_{2p}^{(i)}}{E_{1s}^{(i)}}.$$

- r_{ss}: s-polarization light reflecting to s-polarization
- r_{ps}: s-polarization light reflecting to p-polarization
- t_{ss}: s-polarization light transmitting to s-polarization
- t_{ps}: s-polarization light transmitting to p-polarization

How is This Helpful?

- Boundary conditions require that $A_1P_1=A_2P_2$
  - Can then relate to the r, t coefficients
- Can then determine information about Q, which is information about sample magnetization
  - Paper proposes determining r, t after knowing Q, but the converse is also possible

How is This Helpful: An Example

- Classic example: Light from vacuum impinges upon material with refractive index, N
  - We control the incident light: give it s-polarization
  - $P_1 = \begin{pmatrix} E_{1s}{}^{(i)} \\ 0 \\ E_{1s}{}^{(r)} \\ E_{1p}{}^{(r)} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ r_{ss} \\ r_{ps} \end{pmatrix}, P_2 = \begin{pmatrix} E_{2s}{}^{(i)} \\ E_{2p}{}^{(i)} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} t_{ss} \\ t_{ps} \\ 0 \\ 0 \end{pmatrix}$
  - $A_1 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & \alpha_{1z} & 0 & -\alpha_{1z} \\ 0 & -1 & 0 & -1 \\ \alpha_{1z} & 0 & -\alpha_{1z} & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & \alpha_{2z} & 0 & -\alpha_{2z} \\ 0 & -N & 0 & -N \\ \alpha_{2z}N & 0 & -\alpha_{2z}N & 0 \end{pmatrix}$

How is This Helpful: An Example

- Continuity: $F_1=F_2$ gives the linear equations:

- $$\begin{cases} 1 + r_{ss} = t_{ss} \\ \cos(\theta_1) r_{ps} = \cos(\theta_2) t_{ps} \\ r_{ps} = N t_{ps} \\ \cos(\theta_1)(1 - r_{ss}) = \cos(\theta_2) N t_{ss} \end{cases} \Rightarrow \begin{cases} r_{ss} = \frac{N\cos(\theta_2) - \cos(\theta_1)}{N\cos(\theta_2) + \cos(\theta_1)} \\ t_{ss} = \frac{2N\cos(\theta_2)}{N\cos(\theta_2) + \cos(\theta_1)} \end{cases}$$

- What about r_{ps}, t_{ps}? We need to use Snell's law to further solve:

How is This Helpful: An Example

- $\mathrm{N} = \frac{\sin(\theta_1)}{\sin(\theta_2)}$ gives $\mathrm{t_{ps}}(\cos(\theta_1)\sin(\theta_1) - \cos(\theta_2)\sin(\theta_2)) = 0$
- $\mathrm{t_{ps}}(\sin(2\theta_1) - \sin(2\theta_2)) = 0$ in general requires $t_{ps} = r_{ps} = 0$

- In agreement with what we expect from non-magnetic media: no reason for Kerr rotation which would take s-polarization to p-polarization

How is This Helpful: An Example

- If we started with purely p-polarization, then we get to:
- $$\begin{cases} r_{pp} = \frac{N\cos(\theta_1) - \cos(\theta_2)}{N\cos(\theta_1) + \cos(\theta_2)} \\ t_{pp} = \frac{2\cos(\theta_1)}{N\cos(\theta_1) + \cos(\theta_2)} \\ r_{sp} = t_{sp} = 0 \end{cases}$$
- This equation recovers Brewster's phenomenon: $r_{pp} = 0$ when $\theta_1 = \tan^{-1}(N)$

Practical Uses

- Use to deal with a real situation: more than just a 2-media interaction
  - e.g. lab –> film –> substrate: reflection possible at each interface
  - Requires use of another matrix: *medium propagation matrix* for the film

- Later work generalizes to arbitrary magnetization (not purely polar or longitudinal)
  - Zak, Moog, Liu and Bader, Phys. Rev. B. **43**, 8 (1991)