Micromagnetism and magnetic domain walls

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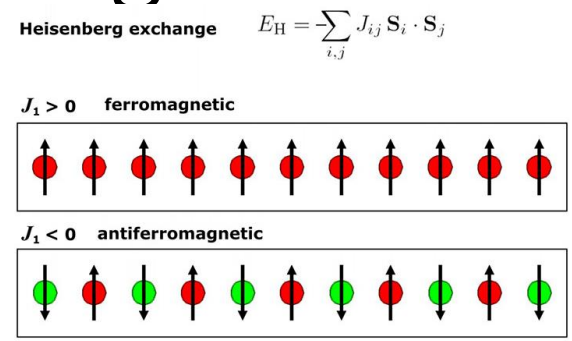
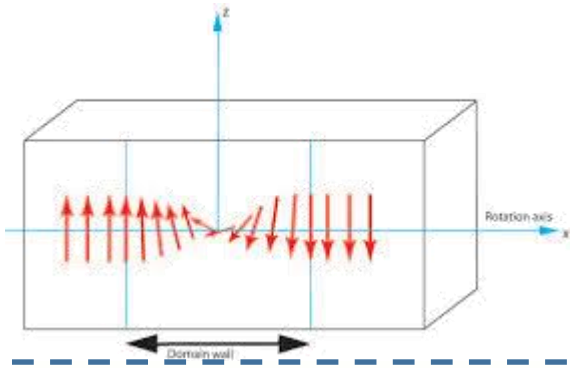
Levels of details for ferromagnets

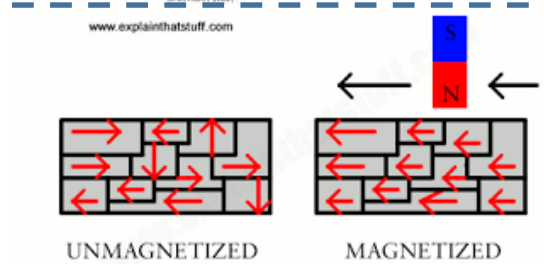
- Atomic level:
  - Exchange interaction that aligns atomic moments $J_{ij}\overrightarrow{S_i} \cdot \overrightarrow{S_j}$



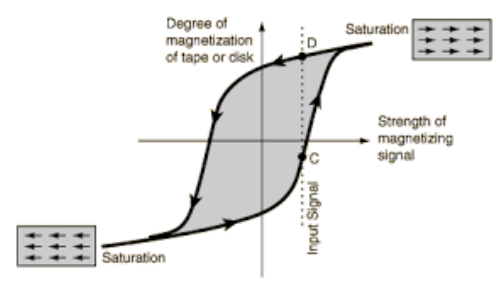
- Micromagnetic level
  - Smear the individual atoms into continuum, see magnetization as a function of position (domain wall)



- Domain level
  - Domains are separated by walls of zero thickness



- Nonlinear level
  - Average magnetization of the entire magnet



Mathematical description

Discrete atomic structure is blended into a continuum

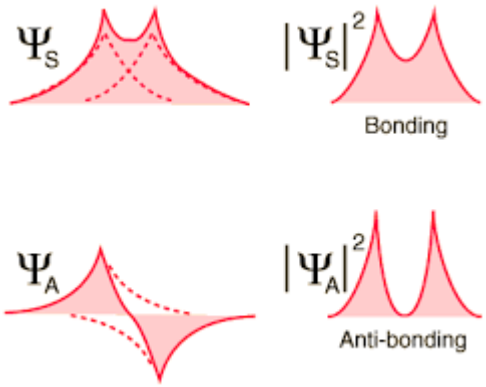
- Exchange interaction energy:

$$W_{ex} = -\Sigma_{i,j} \mathrm{J}\, \overrightarrow{\mathrm{S_i}} \cdot \overrightarrow{\mathrm{S_j}}$$

Becomes

- Exchange energy density

$$w_{ex} = -\lim_{a \to 0} \frac{2J}{a^3} \vec{S}(\vec{\mathrm{r}}) \cdot \vec{S}(\vec{r} + \vec{a})$$



Where a is the primitive lattice constant.

This is in the one-dimensional model.

Mathematical description

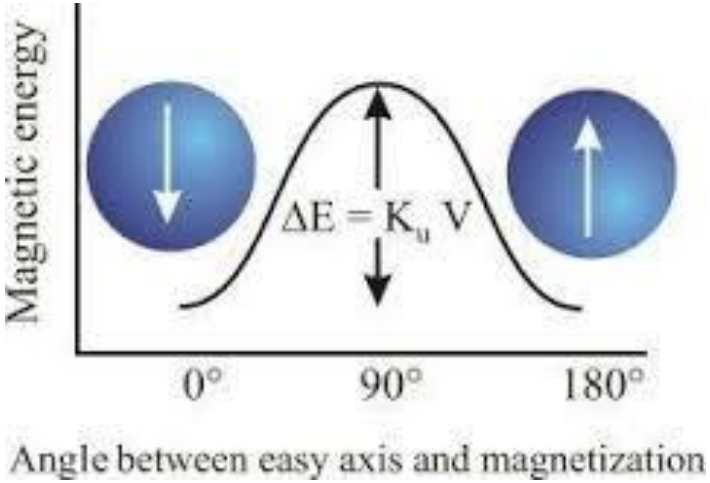
Anisotropy energy (single-ion anisotropy):

$$W_{anis} \propto K_u \Sigma \overrightarrow{\mathrm{S_i}} \cdot \vec{A}$$

Becomes:

Anisotropy energy density (magnetization):

$$w_{anis} = K_u \sin^2 \theta$$



Exchange interaction energy

$$w_{ex} = \lim_{a\to 0} \frac{2J}{a^3} \vec{S}(\vec{r}) \cdot \vec{S}(\vec{r} + \vec{a})$$

$$\vec{\mathrm{S}}(\vec{r}) = \frac{\mathrm{S}_0}{\mathrm{M}_0} \vec{\mathrm{M}}(\vec{\mathrm{r}})$$

$$\vec{\mathrm{S}}(\vec{r} + \vec{a}) = \frac{\mathrm{S}_0}{\mathrm{M}_0} \left[\vec{\mathrm{M}}(\vec{\mathrm{r}}) + \mathrm{a} \frac{\partial \vec{\mathrm{M}}}{\partial x} + \frac{\mathrm{a}^2}{2} \frac{\partial^2 \vec{\mathrm{M}}}{\partial x^2} + \cdots \right]$$

The energy density:

$$w_{ex} = \lim_{a\to 0} \frac{2J}{a^3} \vec{\mathrm{S}}(\vec{r}) \cdot \vec{\mathrm{S}}(\vec{r} + \vec{a}) = \frac{\mathrm{S}_0}{\mathrm{M}_0} \vec{\mathrm{M}}(\vec{\mathrm{r}}) \cdot \vec{\mathrm{S}}(\vec{r} + \vec{a})$$

$$= \lim_{a\to 0} \frac{2J}{a^3} \frac{\mathrm{S}_0^2}{\mathrm{M}_0^2} \vec{\mathrm{M}}(\vec{\mathrm{r}}) \cdot \left[\vec{\mathrm{M}}(\vec{\mathrm{r}}) + \mathrm{a} \frac{\partial \vec{\mathrm{M}}}{\partial x} + \frac{\mathrm{a}^2}{2} \frac{\partial^2 \vec{\mathrm{M}}}{\partial x^2} + \cdots \right]$$

1st term:

$$\vec{\mathrm{M}}(\vec{\mathrm{r}}) \cdot \vec{\mathrm{M}}(\vec{\mathrm{r}}) = \mathrm{constant}$$

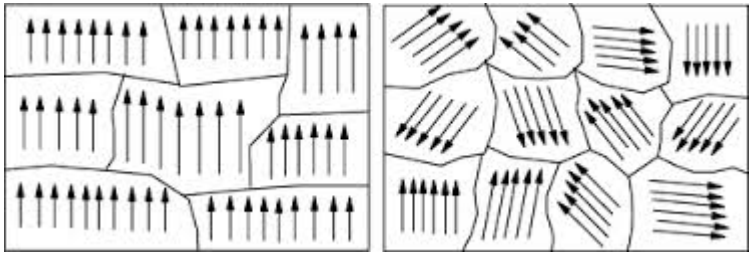
2nd term:

$$\vec{\mathrm{M}} \cdot \frac{\partial \vec{M}}{\partial x} = \frac{1}{2} \frac{\partial M^2}{\partial x} = 0$$

3rd term:

$$\lim_{a\to 0} \frac{2J}{a^3} \frac{\mathrm{a}^2}{2} \frac{\mathrm{S}_0^2}{\mathrm{M}_0^2} \vec{\mathrm{M}}(\vec{\mathrm{r}}) \cdot \frac{\partial^2 \vec{\mathrm{M}}}{\partial x^2} = \lim_{a\to 0} \frac{J}{a} \frac{\mathrm{S}_0^2}{\mathrm{M}_0^2} \vec{\mathrm{M}}(\vec{\mathrm{r}}) \cdot \frac{\partial^2 \vec{\mathrm{M}}}{\partial x^2}$$

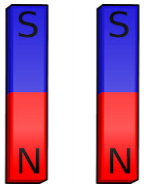
Domain walls: origin



$$W = W_{ex} + W_{anis} + W_H$$

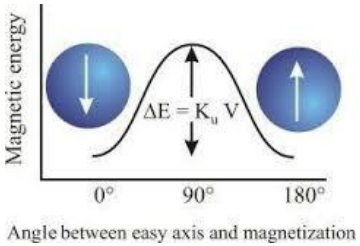
Magnetic static energy from the dipole-dipole interaction:

$$W_H = \int \vec{M} \cdot \vec{H} dV > 0$$
$$\propto N^2$$



$$W_{ex} = -\Sigma J \overrightarrow{S_i} \cdot \overrightarrow{S_j} < 0$$
$$\propto N$$

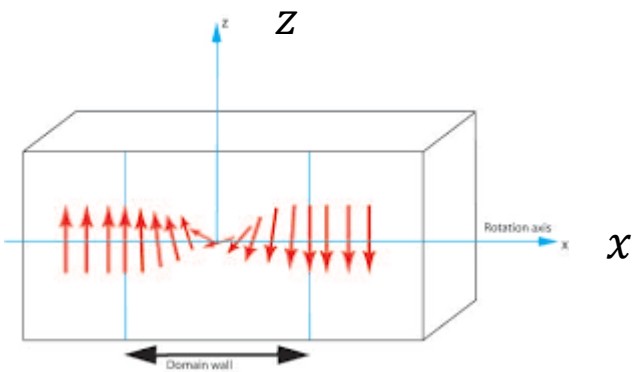
$$W_{anis} \propto K_u \Sigma \left(\overrightarrow{S_i} \cdot \vec{A}\right)^2 < 0$$
$$\propto N$$



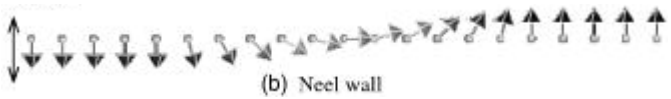
When N $\rightarrow \infty$, W_H wins. So the domains are formed to reduce the total $\vec{M}$.

Domain walls: thickness

180 degree domain



Bloch wall



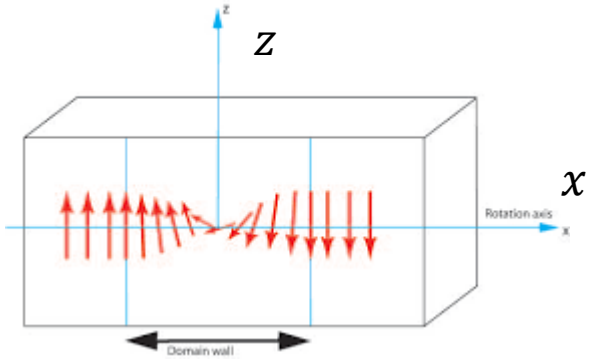
Neel wall

Bloch wall:

$$\vec{M} = M_0(\cos\theta\,\hat{x} + \sin\theta\,\hat{z})$$
$$\theta = \theta(x)$$
$$x = -\infty\colon\ \theta = 0$$
$$x = \infty\colon\ \theta = \pi$$

Domain walls: thickness

$$\vec{M} = M_0(\cos\theta\,\hat{x} + \sin\theta\,\hat{z})$$
$$\theta = \theta(x)$$



Bloch wall

Exchange energy:

$$W_{ex} = \int \frac{J}{a}\frac{S_0^2}{M_0^2}\vec{M}(\vec{r})\cdot\frac{\partial^2\vec{M}}{\partial x^2}dx$$
$$\approx \int \frac{J}{a}\frac{S_0^2}{M_0^2}\vec{M}(\vec{r})\cdot\vec{M}(\vec{r})\left(\frac{d\theta}{dx}\right)^2 dx$$
$$= \int \frac{JS_0^2}{a}\left(\frac{d\theta}{dx}\right)^2 dx$$

Favors smooth change or small $\frac{d\theta}{dx}$

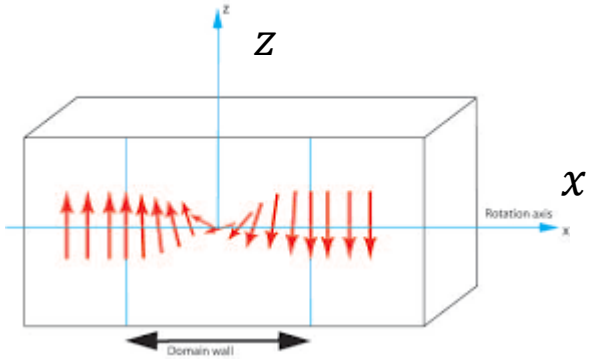
Anisotropy energy

$$W_{anis} = \int K_u \sin^2\theta\, dx$$

Favors abrupt change at the interface.

Domain walls: thickness

$$\vec{M} = M_0(\cos\theta\,\hat{x} + \sin\theta\,\hat{z})$$
$$\theta = \theta(x)$$



Bloch wall

Minimizing

$$\mathrm{W} = W_{ex} + W_{anis} = \int \left[\frac{JS_0^2}{a}\left(\frac{d\theta}{dx}\right)^2 + K_u \sin^2\theta\right] dx$$

Using the variational method:

$$\frac{JS_0^2}{a}\frac{d^2\theta}{dx^2} - \frac{d(K_u \sin^2\theta)}{d\theta} = 0$$

Or

$$\frac{JS_0^2}{a}\left(\frac{d\theta}{dx}\right)^2 = K_u \sin^2\theta$$

$$x = \sqrt{\frac{JS_0^2}{aK_u}} \int \frac{d\theta}{\cos\theta} = \frac{\mathrm{l_w}}{\pi} \ln\left(\tan\frac{\theta}{2}\right)$$

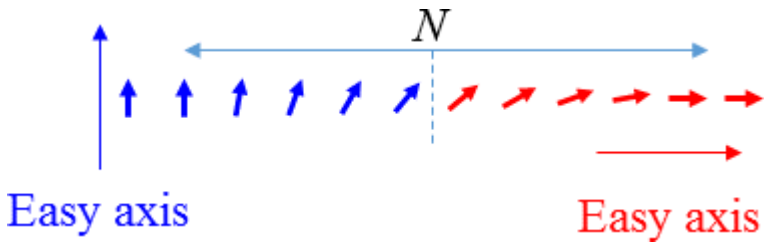
Domain wall width:

$$\mathrm{l_w} = \pi\sqrt{\frac{JS_0^2}{aK_u}}$$

Thicker wall for softer magnets. For iron, this is about **42 nm**.

Applications

In Co3Fe3Nb2 alloys, we found that all of the moments are along all directions.
Given that the particle size is small (60 nm), this could be due to the “domain wall” effect.



Conclusion

- In the micromagnetism, magnetic moments are treated as continuum rather than discrete values.
- This can be used to study the rotation of moments in ferromagnets such as domain walls.