MOKE: Magnetization in Plane of Incidence

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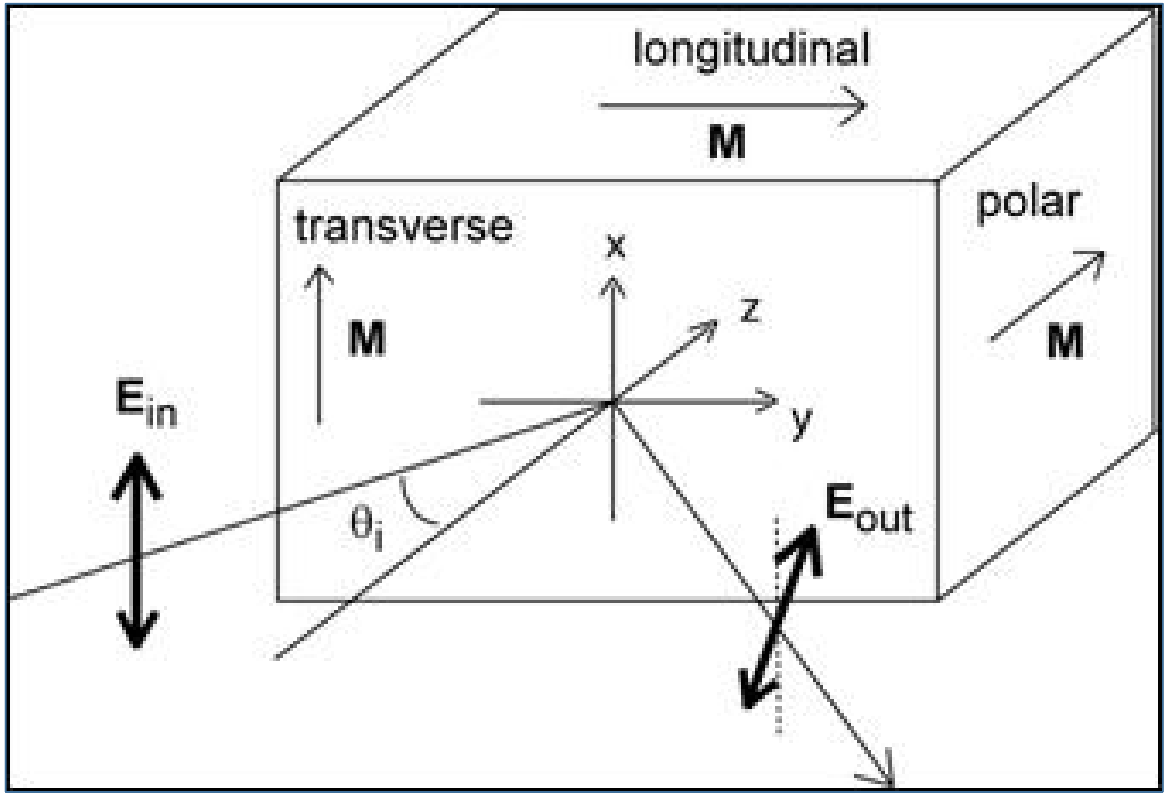
July 1, 2016

Xu Group Meeting

MOKE Geometries

- Polar MOKE
  - Magnetization direction out of plane; incident light at near normal incidence
- Longitudinal MOKE
  - Magnetization direction in plane of surface and in plane of incidence
- Transversal MOKE
  - Magnetization direction in plane of surface and normal to plane of incidence



Fresnel Equation Derivation

- Maxwell equations can be combined into form of a *wave equation*:
  - $\nabla^2 \vec{E}(\vec{r}, t) = \frac{1}{c^2} \underline{\epsilon} \frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2}$, "_" denotes tensor quantity
- Use time- and position-Fourier Transforms on equation:
  - $\mathcal{F}\left[\nabla^2 \vec{E}(\vec{r}, t)\!: \vec{r} \rightarrow \vec{k}, t \rightarrow \omega\right] = -\vec{k} \times \left(\vec{k} \times \vec{E}(\vec{k}, \omega)\right)$
  - $\mathcal{F}\left[\frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2} : \vec{r} \rightarrow \vec{k}, t \rightarrow \omega\right] = \omega^2 \underline{\epsilon}\, \vec{E}(\vec{k}, \omega)$
- Simplify triple product using identity $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$
- $\vec{k}\left(\vec{k} \cdot \vec{E}(\vec{k}, \omega)\right) + \frac{\omega^2}{c^2} \underline{\epsilon}\, \vec{E}(\vec{k}, \omega) = k^2 \vec{E}\left(\vec{k}, \omega\right)$

Fresnel Equation Derivation

- Wave number and index of refraction related
  - $k = \frac{\omega}{c} n$, and therefore $\vec{k} = \frac{\omega}{c} \vec{n}$
- $\frac{\omega^2}{c^2} \vec{n}\left(\vec{n} \cdot \vec{E}(\vec{k}, \omega)\right) + \frac{\omega^2}{c^2} \underline{\epsilon}\, \vec{E}\left(\vec{k}, \omega\right) = n^2 \frac{\omega^2}{c^2} \vec{E}\left(\vec{k}, \omega\right)$
- Eliminate common factor $\frac{\omega^2}{c^2}$, and rewrite first term as "dyadic product" $\vec{n}{:}\,\vec{n}$ (e.g. $n_x n_y$)
- $\left[n^2 \underline{1} - \underline{\epsilon} - \vec{n}{:}\,\vec{n}\right] \cdot \vec{E} = 0$
  - Eigenvalue problem: allowed n^2 values satisfy $\det\left[n^2 \underline{1} - \underline{\epsilon} - \vec{n}{:}\,\vec{n}\right] = 0$

Dielectric Tensor, $\underline{\epsilon}$

- Off-diagonal elements become nonzero when material is magnetized
  - $\underline{\epsilon} = \begin{pmatrix} \epsilon_1 & \epsilon_2 m_z & -\epsilon_2 m_y \\ -\epsilon_2 m_z & \epsilon_1 & \epsilon_2 m_x \\ \epsilon_2 m_y & -\epsilon_2 m_x & \epsilon_1 \end{pmatrix}$
  - Assumes isotropic dielectric and magnetic behavior, no other off-diagonal terms
- (m_x, m_y, m_z) is unit magnetization direction vector
- ϵ_2 is related (but not equal) to magnetization

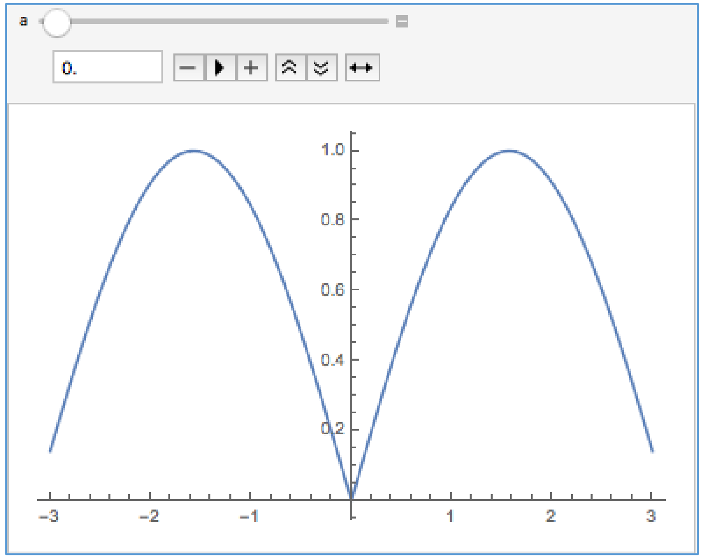
Fresnel Equation Solutions

- Why is transversal MOKE unfavorable for measurement?
  - Explainable from solution to Fresnel equation
- Write $\vec{n} = n(0, \sin\theta, \cos\theta)$
- $$\begin{vmatrix} \epsilon_1 - n^2 & 0 & 0 \\ 0 & \epsilon_1 - n^2\cos^2\theta & \epsilon_2 + n^2\sin\theta\cos\theta \\ 0 & -\epsilon_2 + n^2\sin\theta\cos\theta & \epsilon_1 - n^2\sin^2\theta \end{vmatrix} = 0$$
- Solutions: $n^2 = \epsilon_1$ and $n^2 = \epsilon_1 + \frac{{\epsilon_2}^2}{\epsilon_1}$

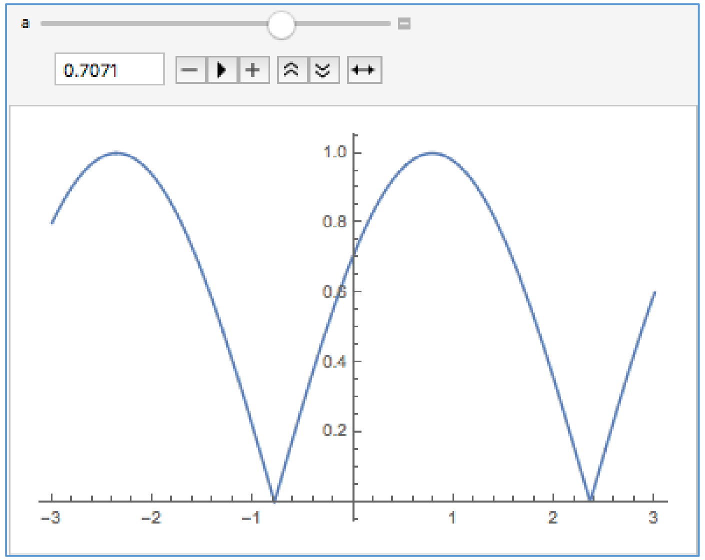
Fresnel Equation Solutions

- General magnetization in yz-plane
- $\begin{vmatrix} \epsilon_1 - n^2 & \epsilon_2 m_z & -\epsilon_2 m_y \\ -\epsilon_2 m_z & \epsilon_1 - n^2\cos^2\theta & n^2\sin\theta\cos\theta \\ \epsilon_2 m_y & n^2\sin\theta\cos\theta & \epsilon_1 - n^2\sin^2\theta \end{vmatrix} = 0$
- Solutions: ${n_\pm}^2 = \epsilon_1 \pm i\epsilon_2\sqrt{1 - \left(m_z\sin\theta - m_y\cos\theta\right)^2}$
  - Correctly reduces to expression for pure polar and longitudinal geometries
  - Terms of order ${\epsilon_2}^2$ and higher are neglected

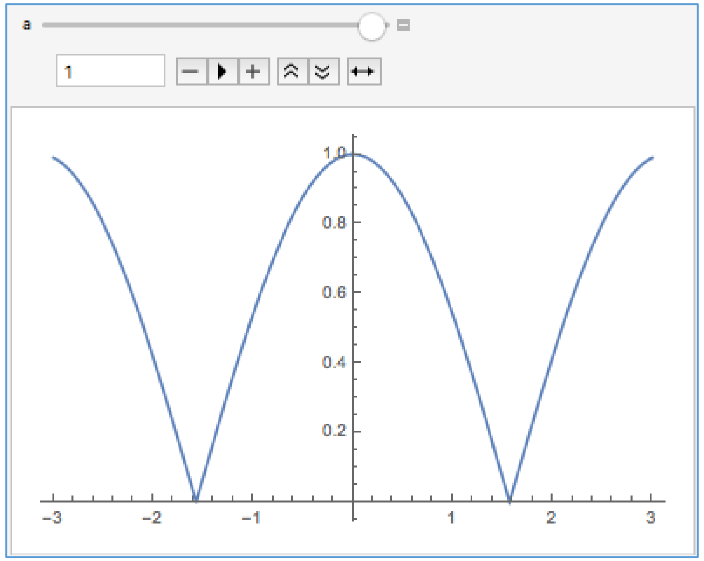
Imaginary part of n^2



$m_z = 0, m_y = 1$ (Longitudinal)
$\theta_{max} = \pi/2$



$m_z = m_y = \sqrt{2}/2$ (Intermediate)
$\theta_{max} = \pi/4$



$m_z = 1, m_y = 0$ (Polar)
$\theta_{max} = 0$