



UNIVERSITY OF NEBRASKA-LINCOLN

Physics & Astronomy

Hall Effect Measurement: Hall Bar and Van der Pauw Geometry

Xiaozhe Zhang

05/20/2016

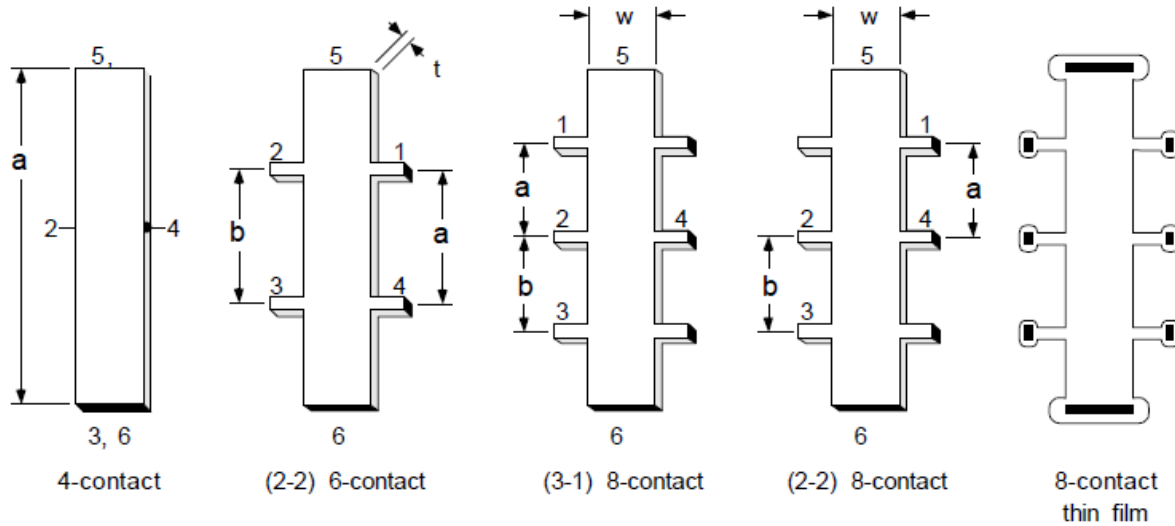
Hall Effect Measurements

Hall effect measurements commonly use two sample geometries:

- Long, narrow Hall bar geometries and
- Nearly square or circular van der Pauw geometries.

Each has advantages and disadvantages. In both types of samples, a Hall voltage is developed perpendicular to a current and an applied magnetic flux.

Hall Bar Geometries



Common Hall Bar Geometries. Sample thickness (t) of a thin film sample = diffusion depth or layer thickness.

Hall bars approximate the ideal geometry for measuring the Hall effect, in which a **constant current density flows along the long axis** of a rectangular solid, perpendicular to an applied external magnetic field.

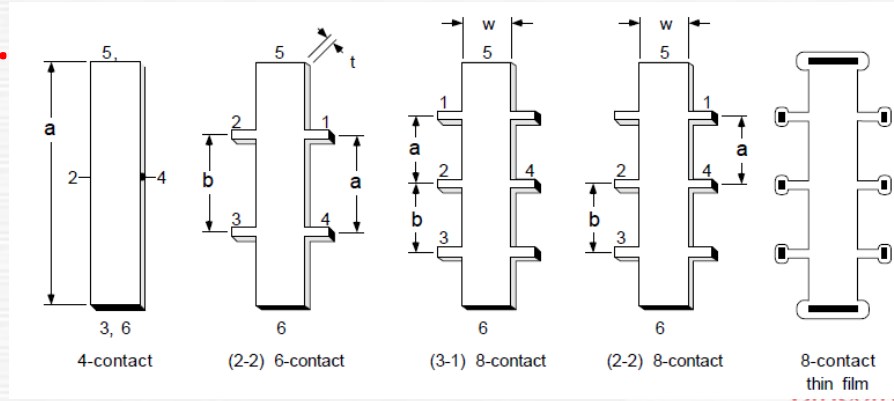
Hall Bar Geometry

The Hall voltage developed across an 8-contact Hall bar sample:

$$V_H = V_{24} = \frac{R_H B I}{t}$$

where V_{24} is the voltage measured between the opposing contacts numbered 2 and 4, R_H is the Hall coefficient of the material, B is the applied magnetic flux density, I is the current, and t is the thickness of the sample

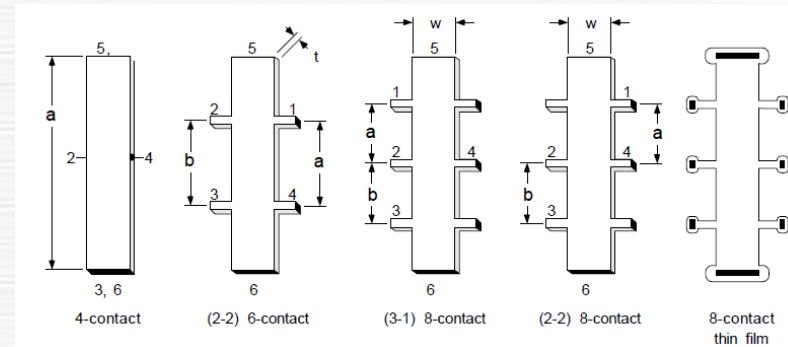
For a given material, increase the Hall voltage by **increasing B and I and by decreasing sample thickness.**



Carrier Type and Carrier Density

The relationship between the Hall coefficient and the type and density of charge carriers can be complex, but useful insight can be developed by examining the limit $B \rightarrow \infty$, when:

$$R_H = \frac{r}{q(p - n)}$$



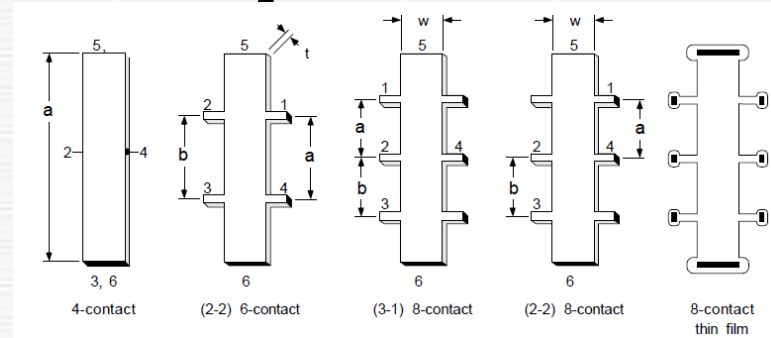
where r is the Hall scattering factor, q is the fundamental electric charge, p is the density of positive and n the density of negative charge carriers in the material.

- For the case of a material with **one dominant carrier**, the Hall coefficient is **inversely proportional to the carrier density**.
- The measurement implication is that the **greater the density** of dominant charge carriers, the **smaller the Hall coefficient** and the **smaller the Hall voltage** which must be measured.
- The **scattering factor r** depends on the scattering mechanisms in the material and typically lies between 1 and 2.

Hall Bar Geometry

Carrier mobility

$$\mu_H = \frac{|R_H|}{\rho}$$



where μ_H is the Hall mobility and ρ is the electrical resistivity at zero magnetic flux density.

The electrical resistivity can be measured by applying a current between contacts 5 and 6 of the sample and measuring the voltage between contacts 1 and 3, then using the formula:

$$\rho(B) = \frac{V_{13}}{I_{56}} \frac{wt}{b}$$

where w is the width and t is the thickness of the Hall bar, b is the distance between contacts 1-3, and B is the magnetic flux density at which the measurement is taken.

Advantages and Disadvantages

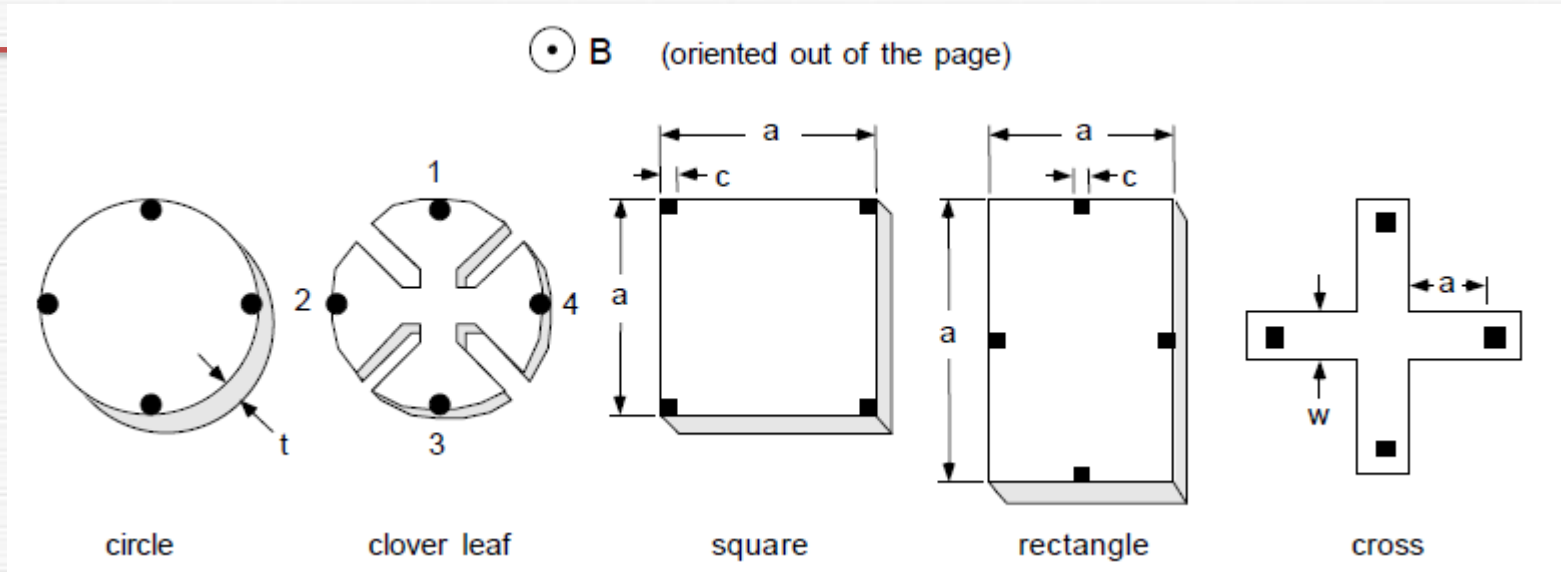
Hall bar is a good geometry for making resistance measurements since about half of the voltage applied across the sample appears between the voltage measurement contacts. For this reason, Hall bars of similar geometries are commonly used when measuring **magnetoresistance or Hall mobility on samples with low resistances**.

Disadvantages of Hall bar geometries include the following:

- A minimum of six contacts to make mobility measurements;
- accuracy of resistivity measurements is sensitive to the geometry of the sample;
- Hall bar width and the distance between the side contacts can be especially difficult to measure accurately.

The accuracy can be increased by making contact **to the sides of the bar at the end of extended arms**. Creating such patterns can be difficult and can result in fragile samples.

Van der Pauw Geometry



Common van der Pauw Sample Geometries. The cross appears as a thin film pattern and the others are bulk samples. Contacts are black.

Calculate the resistivity, carrier concentration, and mobility of an arbitrary, flat sample if the following conditions are met:

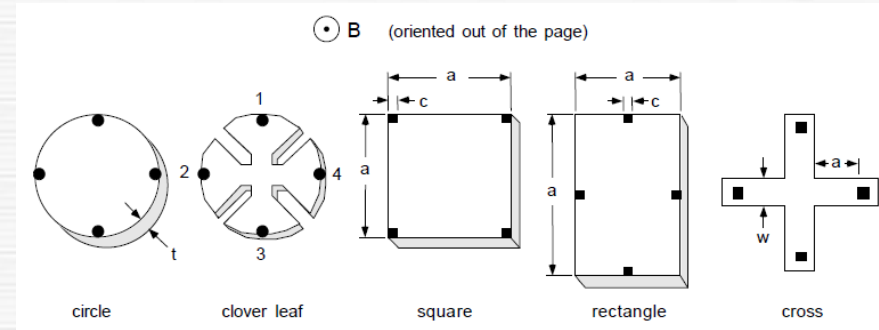
- The contacts are on the circumference of the sample.
- The contacts are sufficiently small.
- The sample is of uniform thickness
- The sample is singly connected (contains no isolated holes).

Van der Pauw Geometry

The resistivity

$$\rho = \frac{\pi t}{\ln(2)} \left(\frac{V_{43}}{I_{12}} + \frac{V_{14}}{I_{23}} \right) / 2$$

where V_{23} is defined as $V_2 - V_3$ and I_{12} indicates the current enters the sample through contact 1 and leaves through contact 2.



Two voltage readings are required with the van der Pauw sample, whereas the resistivity measurement on a Hall bar requires only one. This same requirement applies to Hall coefficient measurement as well, so **equivalent measurements take twice** as long with van der Pauw samples.

F factor for Van der Pauw measurement

The quantity F is the function of the ratio R_r ,

$$R_r \equiv \frac{V_{43}}{I_{12}} \frac{I_{23}}{V_{14}} \equiv \frac{R_{12,43}}{R_{23,14}} \quad \text{or} \quad R_r \equiv \frac{I_{12}}{V_{43}} \frac{V_{14}}{I_{23}} \equiv \frac{R_{23,14}}{R_{12,43}}$$

whichever is **greater**, and F is found by solving the equation:

$$\frac{R_r - 1}{R_r + 1} = \frac{F}{\ln(2)} \operatorname{ar\,cosh} \left\{ \frac{\exp[\ln(2) / F]}{2} \right\}$$

$F=1$ when $R_r=1$, which occurs with symmetrical samples like circles or squares when the contacts are equally spaced and symmetrical. **The best measurement accuracy is also obtained when $R_r=1$.**

Impact of shape and contact state

- Squares and circles are the most common van der Pauw geometries, but contact size and placement can significantly effect measurement accuracy. Others have shown that for square samples with sides of length a and square or triangular contacts of size d in the four corners, **if $d/a < 0.1$, then the measurement error is less than 10%**. The error is reduced by placing the contacts on square samples at the midpoint of the sides.
- The Greek cross has arms which serve **to isolate the contacts from the active region**. When using the Greek cross sample geometry with $a/w > 1.02$, **less than 1% error is introduced**.
- A cloverleaf shaped structure is often used for a patternable thin film on a substrate. The active area in the center is connected by four pathways to four connection pads around its perimeter. This shape makes the measurement much less sensitive to contact size, **allowing for larger contact areas**.

The contact size affects voltage required to pass a current between two contacts. **Ideal point contacts would produce no error** due to contact size, but require an enormous voltage to force the current through the infinitesimal contact area.

Even with square contacts in the corners of a square sample with $d/a < 0.1$, **the ratio of the output to input voltage V_{43}, V_{12} is on the order of 1/10**. Van der Pauw sample geometries are thus much **less efficient** at using the available excitation voltage than Hall bars.

Advantages and Disadvantages

Advantages:

- Only four contacts required.
- No need to measure sample widths or distances between contacts.
- **Simple geometries** can be used.

Disadvantages:

- Measurements take about twice as long.
- Errors due to contact size and placement can be significant when using simple geometries.

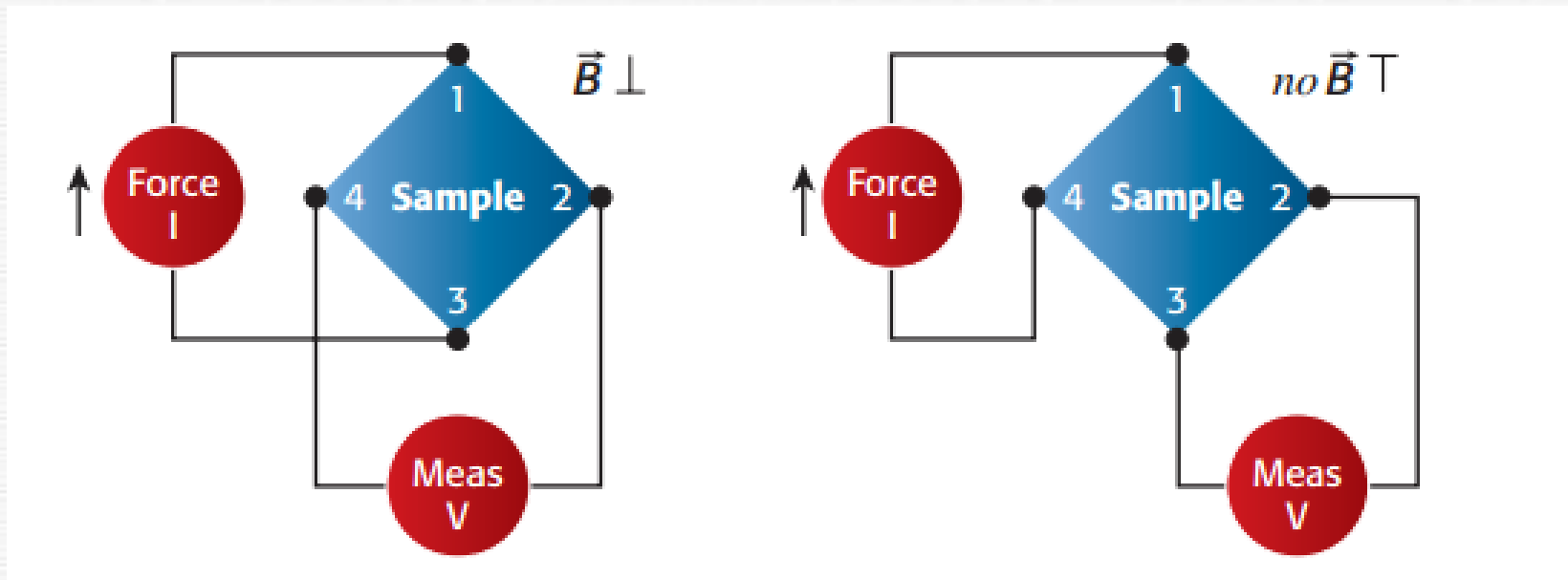
Thank You!

Next topic: Error analysis in general hall effect geometry



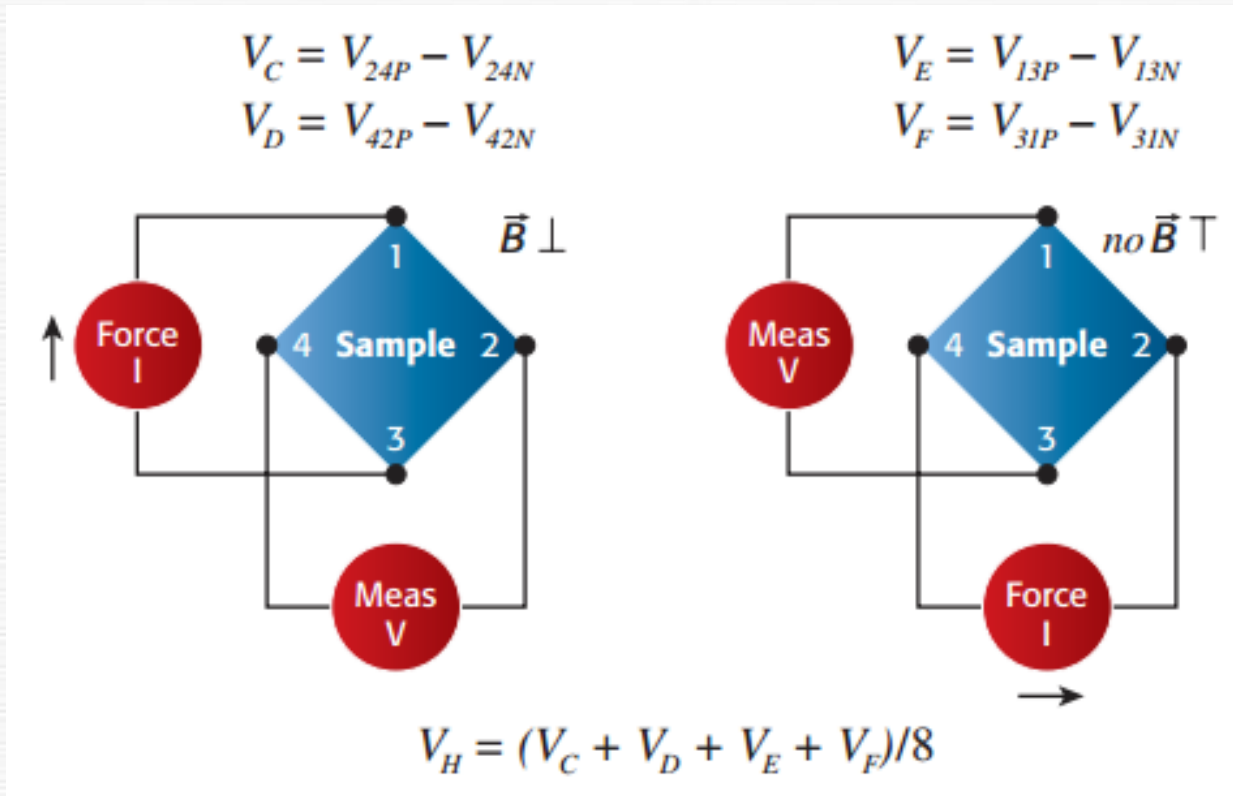
Thank you for your time!

Van der Pauw Geometry



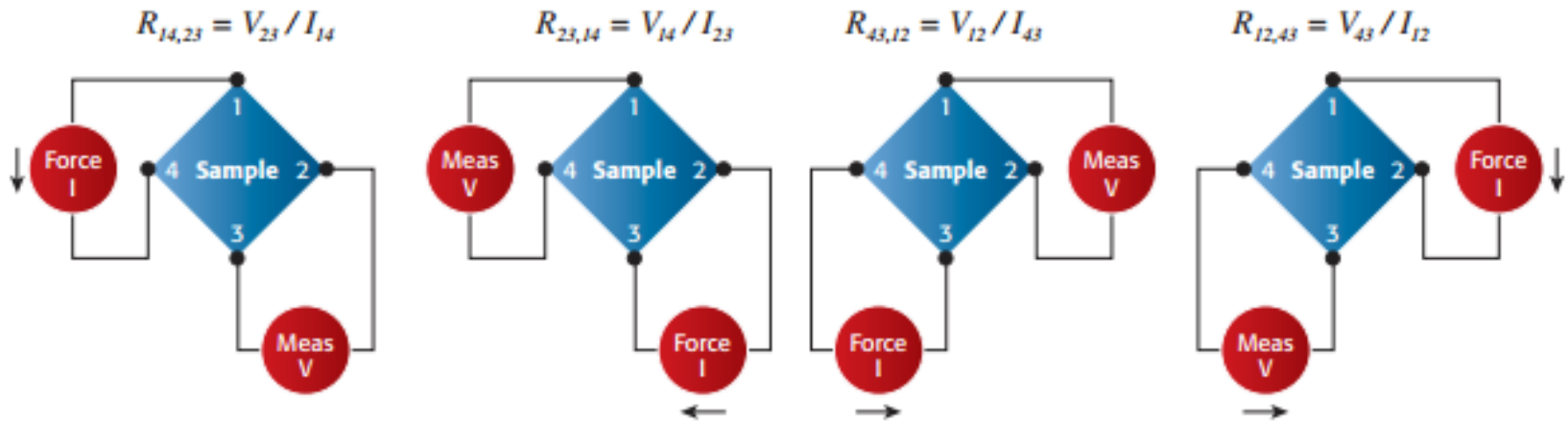
Hall effect voltage vs. van der Pauw resistance measurement configurations.

Van der Pauw Geometry



Compute the Hall voltage with both positive and negative polarity current and with the magnetic field both up and down, and with the two configurations shown. Then average all voltages.

Van der Pauw Geometry



$$R_A = (R_{21,34} + R_{12,43} + R_{43,12} + R_{34,21})/4$$

$$R_B = (R_{32,41} + R_{23,14} + R_{14,23} + R_{41,32})/4$$

$$e^{(-\pi R_s/R_d)} + e^{(-\pi R_d/R_s)} = 1$$

$$\rho = R_s \cdot t$$

Computing average resistivity (ρ) with multiple van der Pauw measurements. Four additional resistance measurements are made with the source current polarity reversed in each of the configurations shown. If $R_A = R_B$, then R simplifies to $\pi R_A / \ln(2)$.